Saturn Moon-Ring Resonances

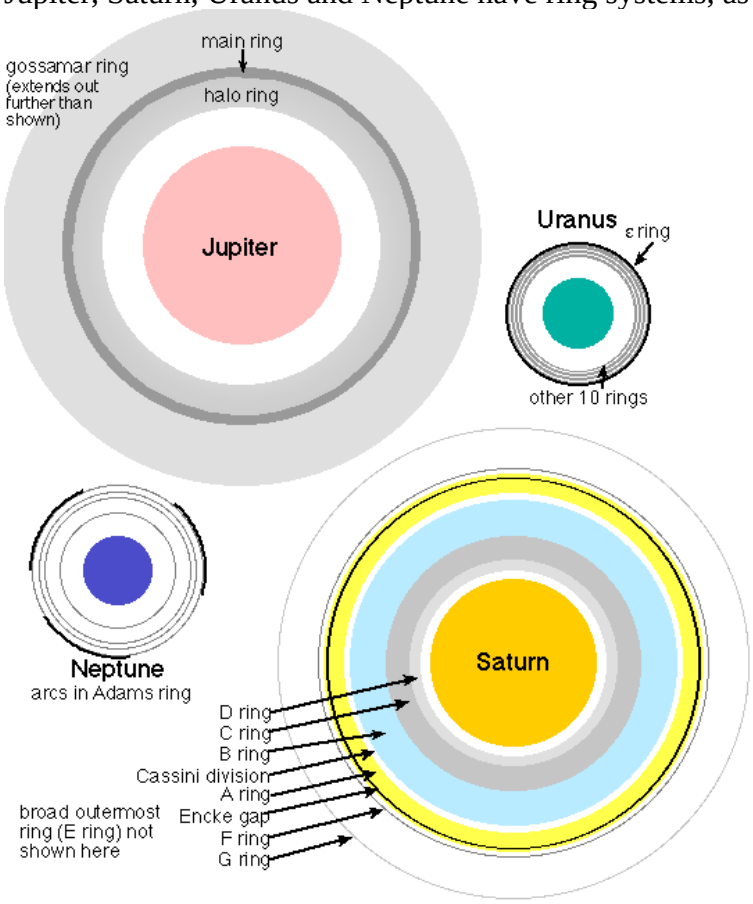
Materials Required
Straight edge

Kepler's Third Law
As applied to low mass objects such as moons or ring particles orbiting Saturn, Kepler's Third Law states that

$$mass_{Saturn} = \frac{(semi-major\ axis)^3}{(period)^2}$$

where the mass of Saturn is in units of the Sun's mass, the semi-major axis is in AU and the period is in Earth years.

Planetary Rings
Jupiter, Saturn, Uranus and Neptune have ring systems, as shown below.



Rings of the jovian planets shown to the same scale

Roché Limit

The Roché Limit is the distance from the center of a planet where the gravitational force is equal to the tidal force.

Take the Earth-Moon system. Each object exerts a tidal force on the other. Pieces of the Moon do not fall off and hit the Earth; bits of the ocean do not fall up to the Moon. In each case, the force of gravity is stronger that the tidal force, so everything stays put.

On the other hand, artificial satellites orbit closer to the Earth than the Roché Limit. Does this mean that they tend to fall apart? Yes! The force of gravity attracting one part of a satellite to another is insufficient to hold the satellite together. It is held together with bolts and nuts, of course.

A large satellite within the Roché Limit for a planet will not have sufficient mechanical strength to survive intact; a tiny object will. A near borderline case might be Saturn's tiny moon Pan, radius 12.8 km, density 0.56 g cm^{-3}, orbit semi-major axis 133580 km, Roché Limit 147000 km

Resonances

If two objects are in orbits around a planet, such that their periods of revolution are in a ratio of small whole numbers, then they will repeatedly exert a pull on each other at the same place (or places) in the orbits, and the orbits will change. In the case of a ring particle, it will be pulled into an orbit that does not have a resonance, and this creates a gap in the ring. Resonances between moons and ring particles can take many forms, and the tiny gaps in the rings of Saturn have not been totally sorted out as yet. It should be noted that every possible resonance does not have a corresponding observed gap. Gaps can take a long time to form.

Experiment

On the next page is an elaborate graph. The y axis is the distance from the center of Saturn, in units of Saturn's equatorial radius. The x axis is a fraction of the period of a moon. Rings and some ring gaps are shown on the far left.

The vertical lines correspond to ratios of small whole numbers. For example, the line labeled 5:3 is at 3/5 = 0.6 of a moon's period. If a ring particle had a period that was 3/5 of a moon, a gap might form at that position in the ring.

The curved lines slanting down towards the left, one line for each moon to be considered, are Kepler's Law calculations giving the size of the orbit that has a given fraction of the period of revolution for the moon.

Example:

The top curve is for the moon Rhea. On the far right is the 1:1 line that represents the moon itself. Nowhere along the Rhea line is there a distance that corresponds to a ring gap. Note, however that the 5:3 position is level with the 1:1 line for Dione. Dione makes 5 revolutions around Saturn while Rhea makes three, and we have discovered a resonance (or very close to one)

Jobs:

Find resonances corresponding to ring gaps or other moons. Make a table on the last page. Answer the questions.

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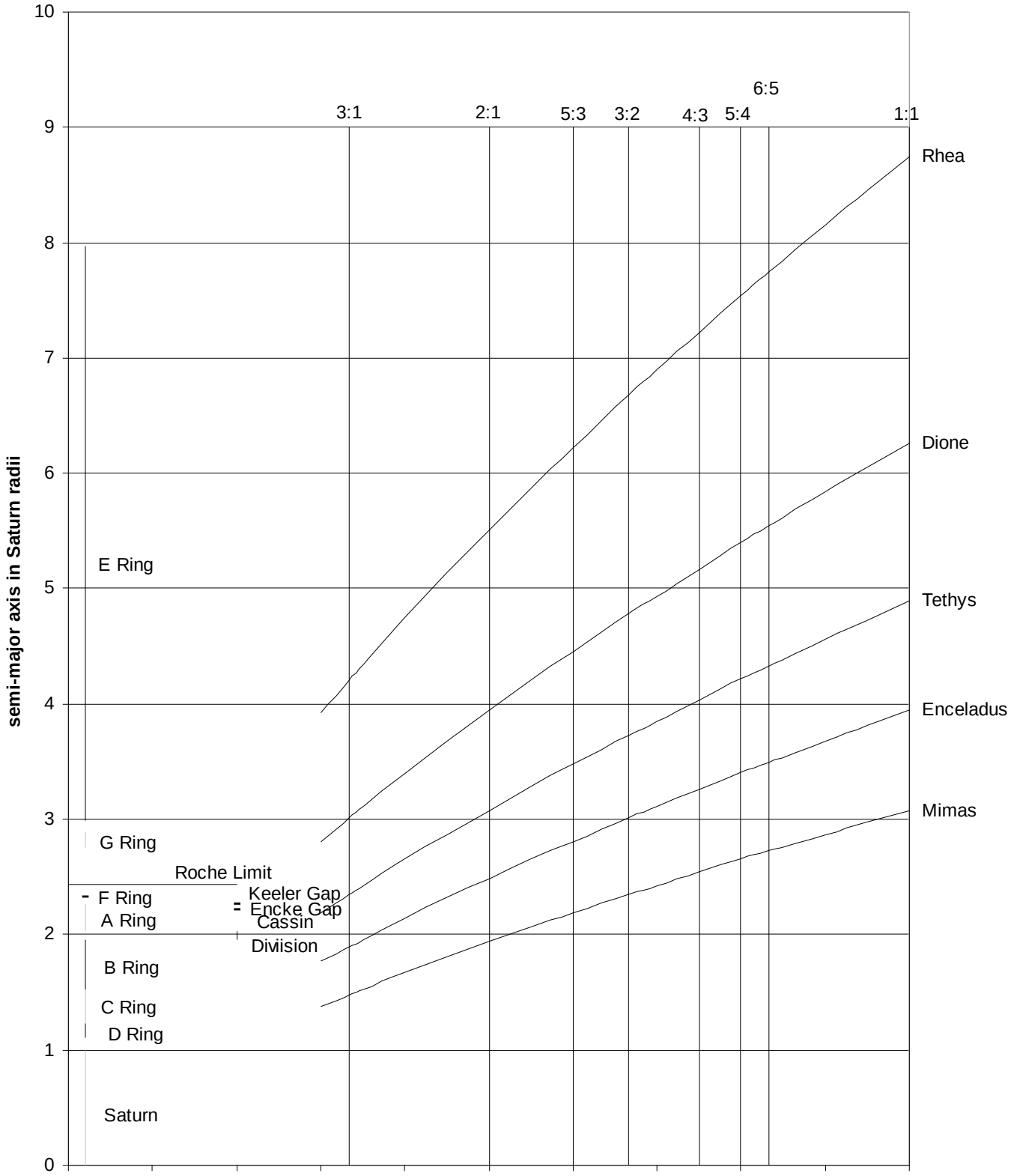

Table of Observed Resonances
Include which moon, the ratio (e.g. 2:1, etc.) and what the resonance is (a ring edge, an observed gap or maybe another moon. All these types are present.

Questions
1) The International Space Station is inside the Roché Limit. If a spacewalking astronaut aboard the International Space Station let go of a tool, what might happen to it?

2) The tenuous G and E rings are outside the Roché Limit. Could the particles in these rings eventually clump together and make a new moon?