Planetary Masses

Objective

Use Kepler's third law to find the mass of a planet.

Introduction

Kepler's third Law, in the form derivable from Newton's Laws of motion is

$$M_1 + M_2 = \frac{(a_1 + a_2)^3}{P^2}$$

where M_1 and M_2 are masses (in units of the solar mass) of two bodies in mutual orbit, a_1 and a_2 (in units of the astronomical unit) are the semi-major axes of the orbits and P is the period of revolution in years.

Except for a quibble about going nearly the speed of light, where Newton's Laws don't hold, we apply Kepler's Laws to the Solar System with awareness that we are making some assumptions, which are good approximations but not perfect:

1) Kepler's Laws assume a two-body system, but the Solar system has many bodies. At least for our Solar System, the mass of the Sun is so much larger than that of any of the planets, that a planet moves mainly under the influence of the Sun, and we can make the approximation that a planet moves <u>only</u> under the influence of the Sun, valid for short periods of time. Over long periods of time, errors accumulate, and planetary orbits billions of years in the future are a matter of speculation.

2) Kepler's Laws assume point masses, or at least perfect spheres, (Newton showed that this amounts to the same thing). Both the Earth and the Moon are lumpy, and the Moon's orbit is slightly but measurably lumpy too.

Since the Sun is by far the most massive body in the Solar system, its own orbit is a tiny wobble and can be neglected. Also, the mass of a planet can be neglected compared to the mass of the Sun. To a very good approximation, the above equation becomes (for our Solar system)

$$1 = \frac{a^3}{P^2}$$

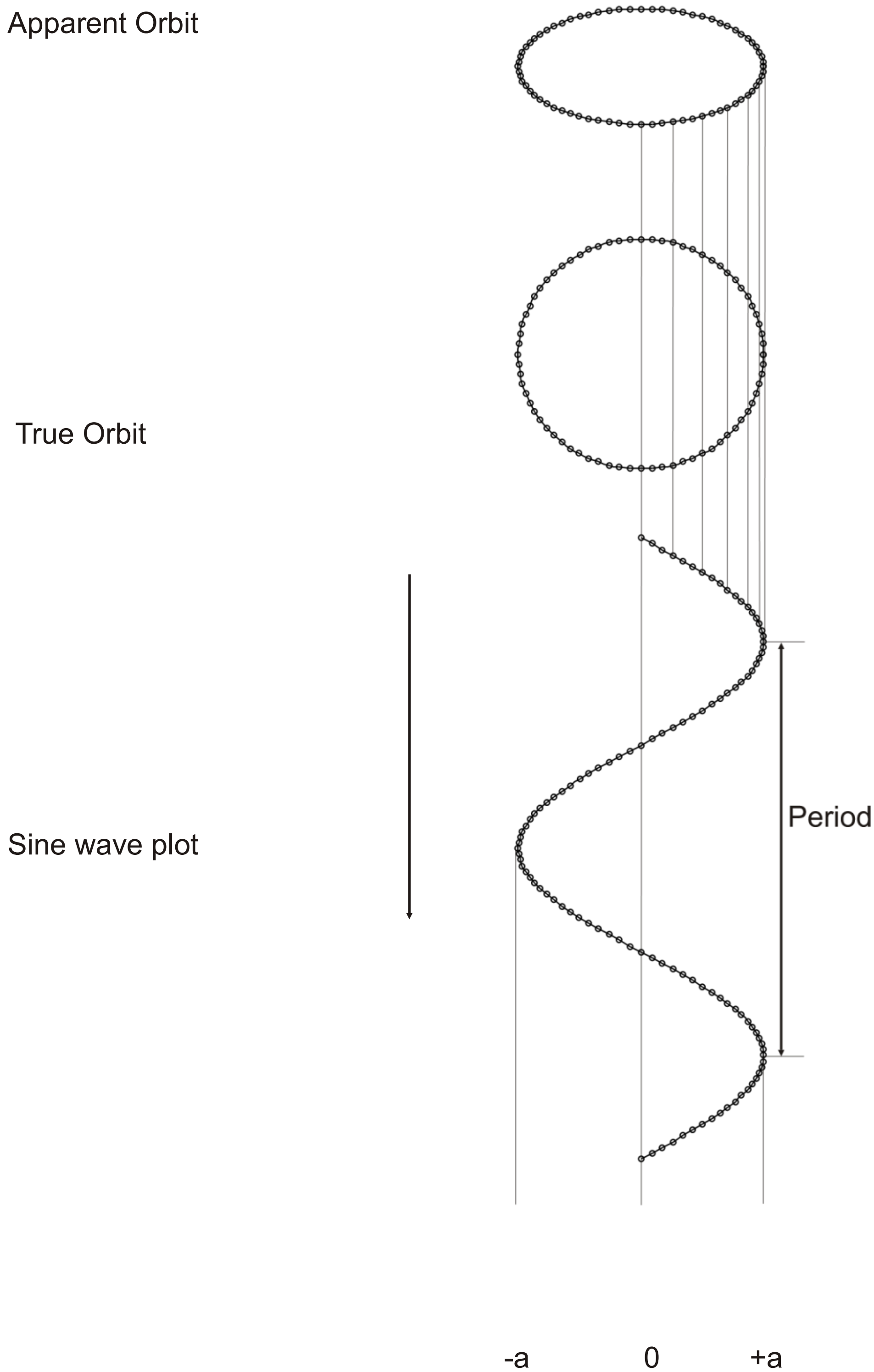
Note that this approximate equation holds true for any planet, or even a pebble. This means that, at this level of approximation, we can get no information at all about the mass of a planet from studying its motion around the Sun. Note the qualifier: It is possible to get a little information about the masses of planets by studying their effects on each other.

There is a better way of obtaining the masses of the planets if the planets have satellites, and that is to apply Kepler's third Law to the planet and its miniature Solar System of moons. If the planet is very much more massive than a moon, we apply Kepler's law in the form

$$M_{planet}\ in\ solar\ mass\ units = \frac{(a\ in\ au)^3}{(Period\ in\ years)^2}$$

The approximation that the planet is very much more massive than the moon is not valid for the Earth-Moon system or the Pluto-Charon system, where we obtain the combined mass of the two bodies. Otherwise, it holds true.

A simplification in data treatment can be made by noting that the orbits of planetary satellites tend to be very nearly circular (Again there is an exception: Nereid, a moon of Neptune). If an orbit is circular, it can appear as a circle, an ellipse, or a straight line, depending upon where we are looking from, but in each case, the largest dimension of the apparent orbit is the diameter of the orbit. Half of this is the quantity that we need for the above equation



For any moon in the solar system (except Nereid) we plot the position of the moon in its progress along the apparent long axis to the left or right of the planet vs time and get a sine wave. The top crest and bottom trough are distance a away from the planet. The period of the revolution is the time between crests. Revolution is counter-clock wise.

Ephemeris Notes

Satellite positions, periods, etc. are for Jan. 1.5, 2000. Perturbations have not been used, except for satellites of Jupiter, and then only the largest terms were used. Consequently, the parameters will be wrong for other dates, with errors increasing with time. For purposes of this experiment, the accuracy is adequate for the range of years 1980 to 2050. Apparent tilts of planets have been ignored, so that the views will not be those seen from Earth for arbitrary dates. A much more accurate solar system simulator is available at http://space.jpl.nasa.gov

Procedure

1) Start the program KEPLER

2) Enter your Name(s)

3) From the File menu, select Planet, and then select which planet is to be studied, according to your instructor's directions.

4) Select a date and time for starting the simulation (your birthday?). The simulation will then run for a length of time which varies with the planetary system. When done. the message "Simulation Halted" will be displayed.

5a) If a printer is available: Click on the Print menu to print a sine wave plot. Repeat to print a second copy if you have a partner.

5b) If a printer is not available: Click on the Measure menu. This opens a new window that displays the cursor position on the graph. Enter your results on the data sheets provided.

6) From the File menu, select Exit to quit the program.

7) For each moon, find the semi-major axis a and the period P from the printed graph. Enter these quantities on the print out or data sheets.

8) For each moon, calculate the planetary mass in solar mass units. Enter these quantities on the print out or data sheets. Calculations are to be done on the last page.

$$M_{planet} \text{ in solar mass units} = \frac{(a \text{ in au})^3}{(\text{Period in years})^2}$$

9) Answer the questions on page 6. Hand in the question sheet and the print out or data sheets. Hand in the calculations. Circle your own name on the print out.

Mars Notes

There are two moons, and both are very small. The orbital periods are very nearly in a ratio of 4:1. The phases of Mars are not shown, even though Mars sometimes appears quite gibbous.

Jupiter Notes

There are sixty-two confirmed moons as of Nov. 2004, and reports of many more. Some don't have official names yet. The four moons discovered by Galileo in 1610 are shown in the simulation. Io and Europa are Earth Moon sized, while Ganymede and Callisto are a bit larger than the planet Mercury.

Saturn Notes

There are thirty-one confirmed moons as of Nov. 2004, and reports of more. Only the first five large moons used for the simulation. The largest moon, Titan, is omitted. Saturn's north pole is shown as tilted away from us. The South pole is visible instead, and the moons appear to move clockwise.

Uranus Notes

There are twenty-four confirmed moons as of Nov. 2004. The five largest moons are used in the simulation. Uranus and the orbits of these moons are inclined by about 90 degrees to the ecliptic.

Neptune Notes

There are thirteen confirmed moons as of Nov. 2004. Two are used for the simulation. Triton has a retrograde motion. Nereid has a highly eccentric orbit (e =0.7512), and calls for special treatment in the simulation. The graph that is printed is for the true orbit, not the orbit as it appears from Earth. The plot is not a sine wave. The amplitudes are a(1-e) on one side and a(1+e) on the other. You will have to find both amplitudes, add them and divide by two to get the value of a.

Pluto Notes

There is only one large moon, Charon. Judging by its radius (593 km), it is not a good bet that its mass is tiny compared to that of Pluto. What you will actually measure is the total mass, not the mass of Pluto alone, which remains poorly determined.

Questions

Name ____ Date _____________________

1) In spite of having no natural moon, the mass of Venus is known very accurately (2.447838×10^{-6} solar masses). Speculate on how this was done.

2a) How would you go about finding the separate masses of Pluto and Charon?
This has been done. The reference is Null, G. W. and Owen, W. M., “Charon/Pluto Mass Ratio Obtained with HST CCD Observations in 1991 and 1993", Astronomical Journal 111, 1368. Their results were

Pluto	1.3×10^{22} kg = 6.4×10^{-9} solar masses
Charon	1.9×10^{21} kg = 9.6×10^{-10} solar masses

2b) Add the masses of Pluto and Charon, taking significant figures into account. Note that you cannot get the correct result with your calculator. Is the result significantly different from the mass of Pluto alone?

3) What other systems might the procedure employed in the experiment be applied to?

Data

Name ____________________ Partner's Name ____________________ Date ______

These data sheets are needed when a printer is not available. Moons are tabulated in order of increasing a.

Mars

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Phobos					
Deimos					

Jupiter

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Io					
Europa					
Ganymede					
Callisto					

Data

Name ____________________ Partner's Name ____________________ Date ______

Saturn

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Mimas					
Enceladus					
Tethys					
Dione					
Rhea					

Data

Name ____________________ Partner's Name ____________________ Date ______

Uranus

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Miranda					
Ariel					
Umbriel					
Titania					
Oberon					

Neptune

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Triton					
Nereid					

Data

Name ______________________________ Date ____________________

Pluto

Moon	a (AU)	P (years)	a^3	P^2	$M = \frac{a^3}{P^2}$
Charon					

Calculations