EXTRA-SOLAR PLANETS

OBJECTIVE
Study the current methods of discovering extra-solar planets and their limitations.

INTRODUCTION
Three methods of discovering extra-solar planets have been successful:
1) Direct Imaging
This is very hard to do, because we are trying to see a faint planet against the glare of the star
2) Measure the Doppler shift of a star as it wobbles in response to a planet's motion.This was first done in 1995 and hundreds of planets have been found this way. In the absence of further information, only the minimum mass of the planet can be found.
3) Measure the dimming of the light from a star as the planet moves in front of it. The dimming is very small, and requires very accurate measurements. As of Aug. 1, 2012, the Kepler Program has discovered some 2000 planet candidates. An example of what we are looking for that we can actually see up close is shown below.

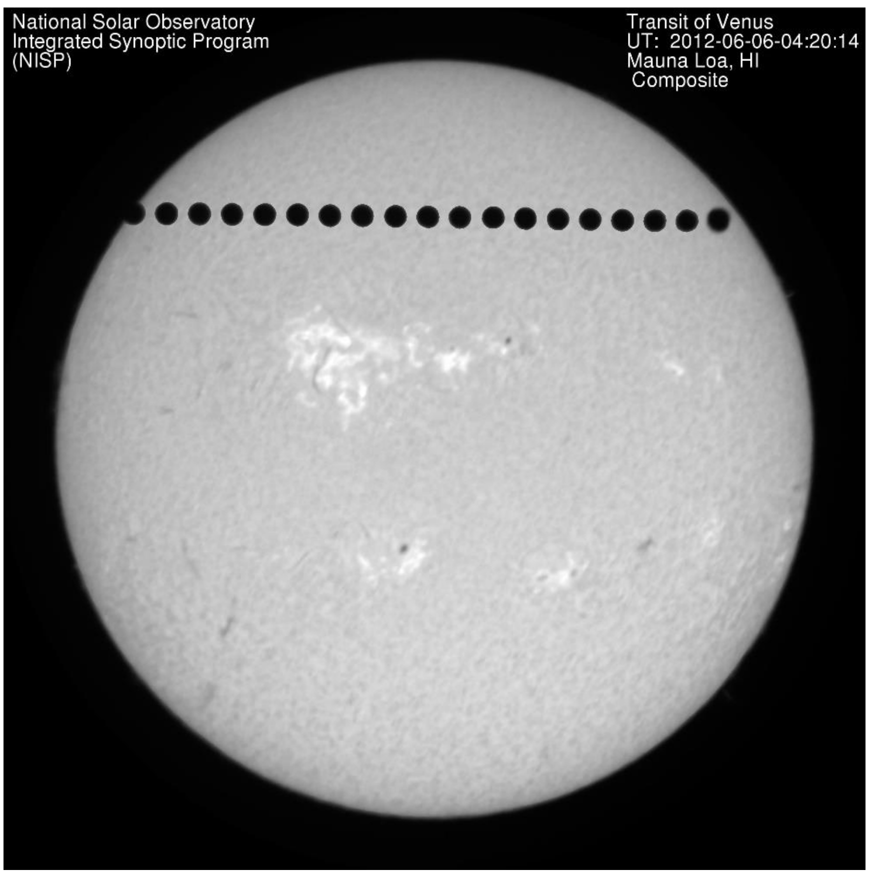


Figure 1. Venus transit, June 5-6, 2012

THE KEPLER PROGRAM
Kepler is a space observatory launched 3/7/09. It has a telescope with a 0.95 m aperture and a wide field of view. It continuously monitors about 150,000 stars. Kepler's single instrument is an array of CCD's that are designed to measure the light from each star to a precision of 20 parts per million. The actual precision is about 29 parts per million; this disparity is apparently due to an underestimation of the rapid fluctuation in the brightness of the stars themselves. The target area is in the constellations of Cygnus, Lyra and Draco. The field is just outside the disk of the Milky Way.

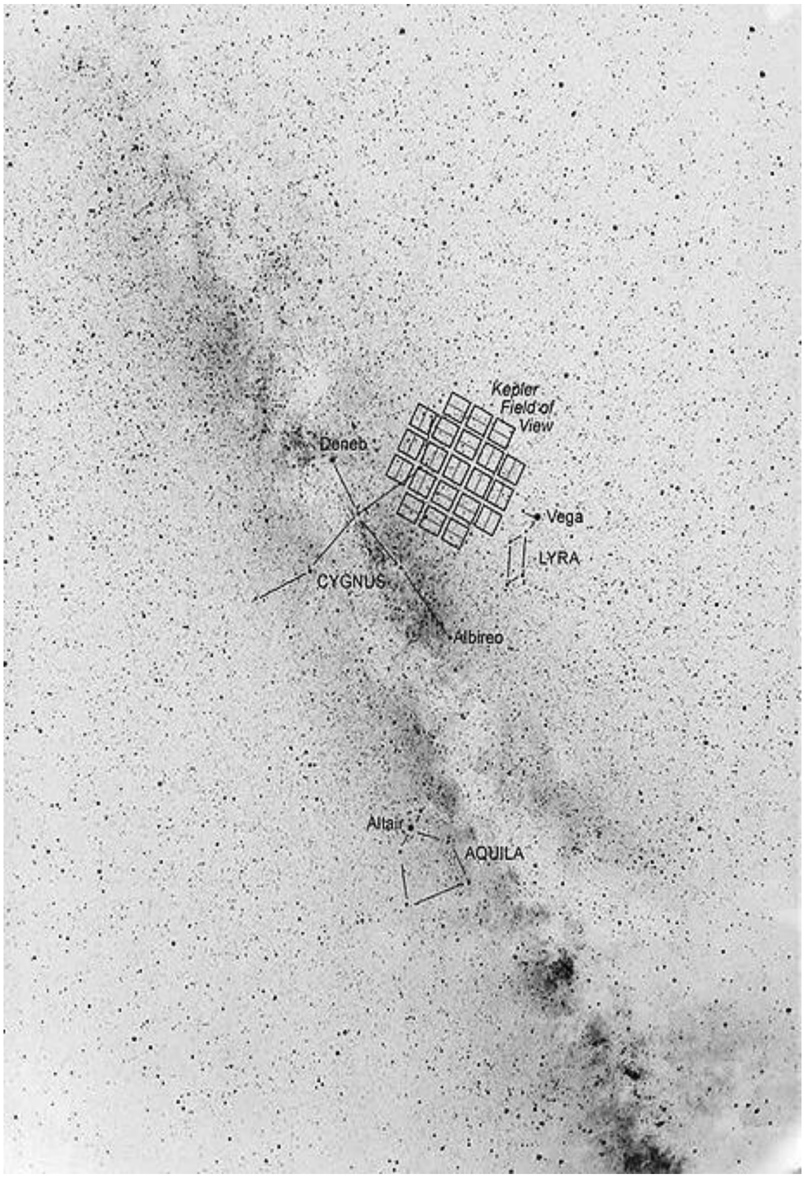


Figure 2. Kepler's field of view.

HIRES
The HIRES spectrometer, installed on the Keck-1 telescope provides Doppler shifts with an accuracy of about one meter per second, and is a technological advance that makes measurements of small star wobbles possible. At this high resolution, fluctuations due to flares and vibrations have to be taken into account.

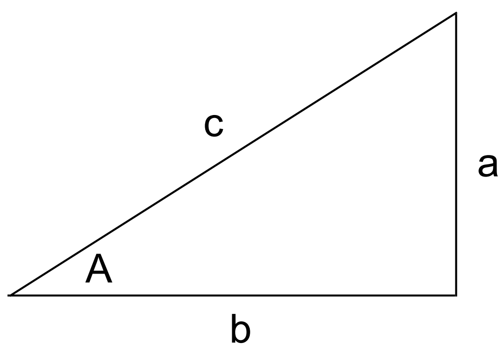
MATH
The ratio of the circumference of a circle to its diameter is π (pi) and is approximately equal to 3.14159

$\text{Area of a circle} = \pi r^2$

$\text{Area of a sphere} = 4\pi r^2$

$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$

Angles are measured in degrees or in radians. One circle = 360 degrees = 2 π radians.



$\sin A = \frac{a}{c}$

$\cos A = \frac{b}{c}$

Also needed are the inverse functions arcsin and arccos, which convert sin and cos values back into angles.

In EXCEL, the following functions are needed:

PI()	π
SIN(x)	sine of the angle x in radians
COS(x)	cosine of the angle x in radians
ASIN(x)	arcsin function; result is in radians
ACOS(x)	arccos function; result is in radians.

Scientific notation presents a value as a number between one and ten times ten raised to an exponent. For example, $123.45 = 1.2345 \times 10^2$

Significant figures is a method for reporting a value and its approximate uncertainty. For example, 123.45 has five significant figures, with the value of the next decimal place being unknown. Confusion is avoided by using scientific notation.

When calculations are performed with measured values, the errors in the measured quantities must be propagated through the calculation. Only the simplest mathematical operations will be considered here. When multiplying or dividing numbers, the result has the minimum number of significant figures of the operands. For example,

$1.24 \times 5.678 = 7.04072$ becomes 7.04

Addition is done the same way, except that the decimal places are placed over one another.

Subtraction is messy, in that one can loose significant figures when the numbers are close together. For example,
$5.678\times10^4 - 5.602\times10^4 = 0.076\times10^4 = 7.6\times10^2$ with only two significant figures.

An error can be defined as a measured value minus a real value. This is not practical, as the real value is not known.

A relative error is the error divided by the real result. It may be converted to a % by multiplying by 100 or parts per million by multiplying by one million.

In the first system to be studied, the Sun-Earth system viewed from 100 parsecs away, we actually have a situation much like that above. The parameters for this system are known to a high degree of precision, and we can use error = measured value minus precise value.

The second system to be studied, Kepler 10b uses real measurements, and there is no precise value available. For a large number of measurements that have random fluctuations, the estimated result can be taken as the average of the measurements and the estimated error can be obtained from the magnitudes of fluctuations. A narrow spread of measurements or more measurements yields a more precise result.

NOTES ON CALCULATIONS

The calculations you will perform are algebraically correct, but numerically poor, since the measured values have large uncertainties. A proper calculation computes everything at once and treats the uncertainties properly. As a result, your calculated values will not be exactly the same as the literature values, although they should agree within the uncertainties in the literature.

PHYSICS

The units that we will be using is the MKS (meter kilogram second) system. The abbreviations are m, km and s. An additional unit is power, which is energy per unit time and is measured in Watts (abbreviation W).

The emission from a blackbody depends on its size and temperature. If something (like a star) is not exactly a blackbody, we use an effective temperature, T_{eff}, which won't be exactly the same as the real temperature. Flux is the power radiated (or received) in units of Watts per square meter. Stefan's law (text, chapter 3) is
$$F\left(W\,m^{-2}\right) = \sigma T^4$$
where
σ (sigma) is the Stefan-Boltzmann constant and T (or T_{eff}) is the temperature in degrees Kelvin.

DEFINITIONS

i = the inclination angle. This is the angle that an orbit makes with the plane of the sky. If i = 0, the orbit is seen face on; if i = 90 degrees, the orbit is seen edge on. The inclination angle has to be fairly close to 90 degrees in order to observe eclipses.

a = the semi-major axis of the planets orbit, measured in meters. If the eccentricity = 0, the case used here, a is the radius of the orbit.

P = the period of revolution in seconds (how long the planet takes to go around the star).

M_1 = mass of the primary (the star) in kilograms.

M_2 = mass of the secondary (the planet) in kilograms.

R_1 = radius of the primary in meters.

R_2 = radius of the secondary in meters.

ω (omega) = the position angle of a planet in its orbit, with 0 representing mid-eclipse.

d = distance to the star in meters.

For an eclipse
First contact is the start of the partial eclipse
Second contact is the start of the total eclipse
Third contact is the end of the total eclipse
Fourth contact is the end of the partial eclipse

F_u = the flux received at the detector for the un-eclipsed star.
F_e = the flux received at the detector for the eclipsed star.

MORE PHYSICS
A star's total power is found from the flux at the star's surface multiplied by the surface area of the star. Assuming that space is perfectly clear, the star's total power is undiminished: let it spread out over the surface of a sphere of radius d.

$$\text{totalpower} = \left(\sigma T^4\right) \times \left(\pi R_1^2\right) = (\text{measuredflux}) \times \left(4\pi d^2\right)$$

The point here is that if T and d are known, R1 can be found (text, chapter 17)
If a star is uniformly bright, we get the flux ratio eclipsed:uneclipsed

$$\frac{F_e}{F_u} = \frac{\pi R_1^2 - \pi R_2^2}{\pi R_1}$$

Look back at the Venus transit picture on the first page. Note that the edge of the Sun is relatively dim. This is called limb darkening, and it occurs whenever an atmosphere is present. This has to be compensated for. It's a messy calculation and I have done it for you.

After compensating for limb darkening, it is easy to pick out the eclipse contacts from the light curve.
Let
$t_0 = 0$ = time of mid eclipse.
t_3 = time of third contact.
t_4 = time of fourth contact.
Then

$$\omega_3 = \frac{2\pi t_3}{P}$$

$$\omega_4 = \frac{2\pi t_4}{P}$$

The geometry then gives equations that can be solve simultaneously for a and sin i

$$\cos\omega_3 = \frac{\sqrt{1-\left(\frac{R_1 - R_2}{a}\right)^2}}{\sin i}$$

$$\cos\omega_4 = \frac{\sqrt{1-\left(\frac{R_1 + R_2}{a}\right)^2}}{\sin i}$$

The solutions (for positive a) are

$$a = \sqrt{\frac{\cos^2\omega_3\left(R_1+R_2\right)^2 - \cos^2\omega_4\left(R_1-R_2\right)^2}{\cos^2\omega_3 - \cos^2\omega_4}}$$

$$\sin i = 2\sqrt{\frac{R_1 R_2}{\cos^2\omega_3\left(R_1+R_2\right)^2 - \cos^2\omega_4\left(R_1-R_2\right)^2}}$$

Something known as the mass function is (for eccentricity =0)

$$f(M) = \frac{M_2^3 \sin^3 i}{\left(M_1+M_2\right)^2} = \frac{P K^3}{2\pi G}$$

where
K is the velocity (semi) amplitude of the primary. We also have Kepler's third law, which can be written as

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{G\left(M_1+M_2\right)}$$

Solving the above set of two equations gives

$$M_1 = \frac{2\pi\left(2a^3\pi\sin i - a^2 K P\right)}{G P^2 \sin i}$$

$$M_2 = \frac{2a^2 K\pi}{G P \sin i}$$

EXPERIMENT
Two systems are to be investigated.

The first system consists of the Sun plus the Earth in a circular orbit with no other planets. It is to be viewed from a planet inhabited by the LGA (little green astronomers), 100 parsecs away and close enough to the plane of the ecliptic so that eclipses can be observed.

Figure 3. LGA
The point of doing this sort of simulation is that calculated values can be compared with real ones.

The second system is real. The star is 2MASS 19024305+5014286 and is shown in the photograph below.

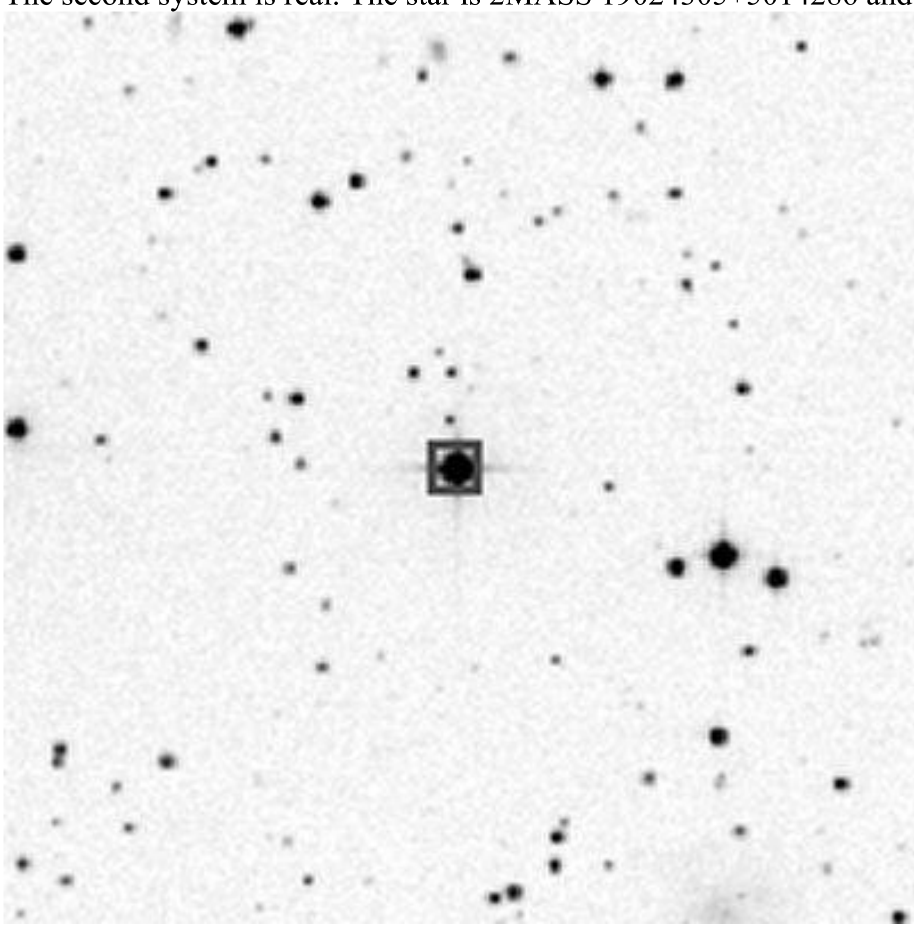


Figure 4. 2MASS 19024305+5014286 has a box around it. The field is 5’ on a side.

There are no really bright stars in the immediate vicinity of the star of interest, which is good because such a star could mess up the measurements. There were 11,000 measurements of brightness for this system using Kepler. Satisfying conservative requirements awards this system with a confirmation as a planet, which has been assigned the name Kepler-10b. There are strong indications of at least one more planet, but it has not yet been verified, and there are no Doppler shifts at the required period observed.

Here is the Kepler 10b data (Natalie M. Batalha, *et. al*., The Astrophysical Journal, 729 (2011) 27)

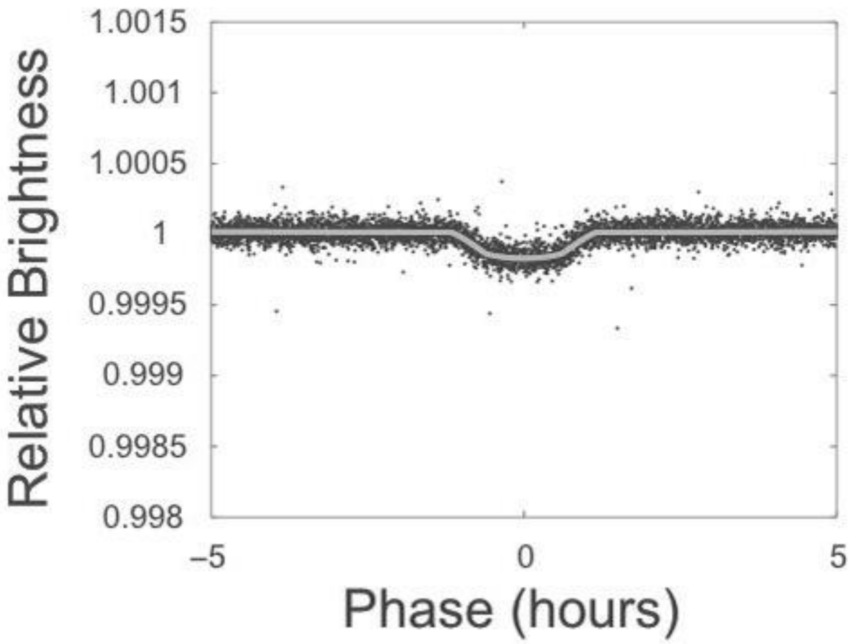


Figure 5. This is the light curve for Kepler 10b. The lighter portion in the center of the noise represents an average. There is no correction for limb darkening applied as yet.

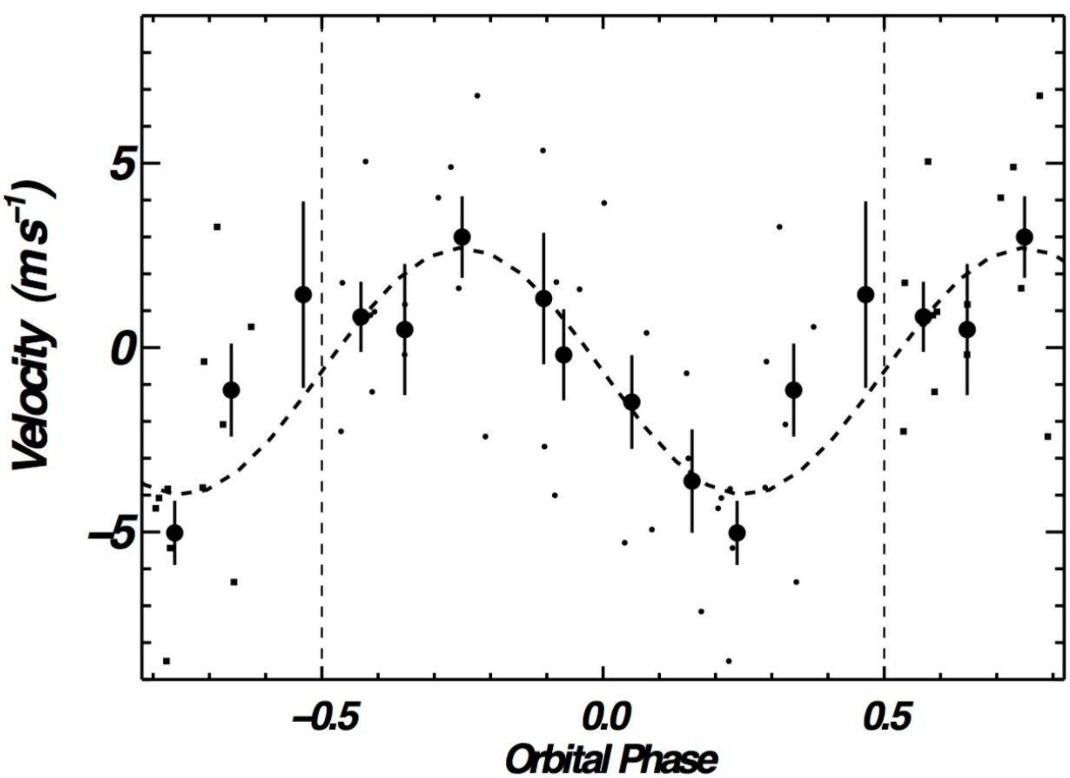


Figure 6. Kepler 10b Doppler curve. The small dots are measurements; the large dots are averages.

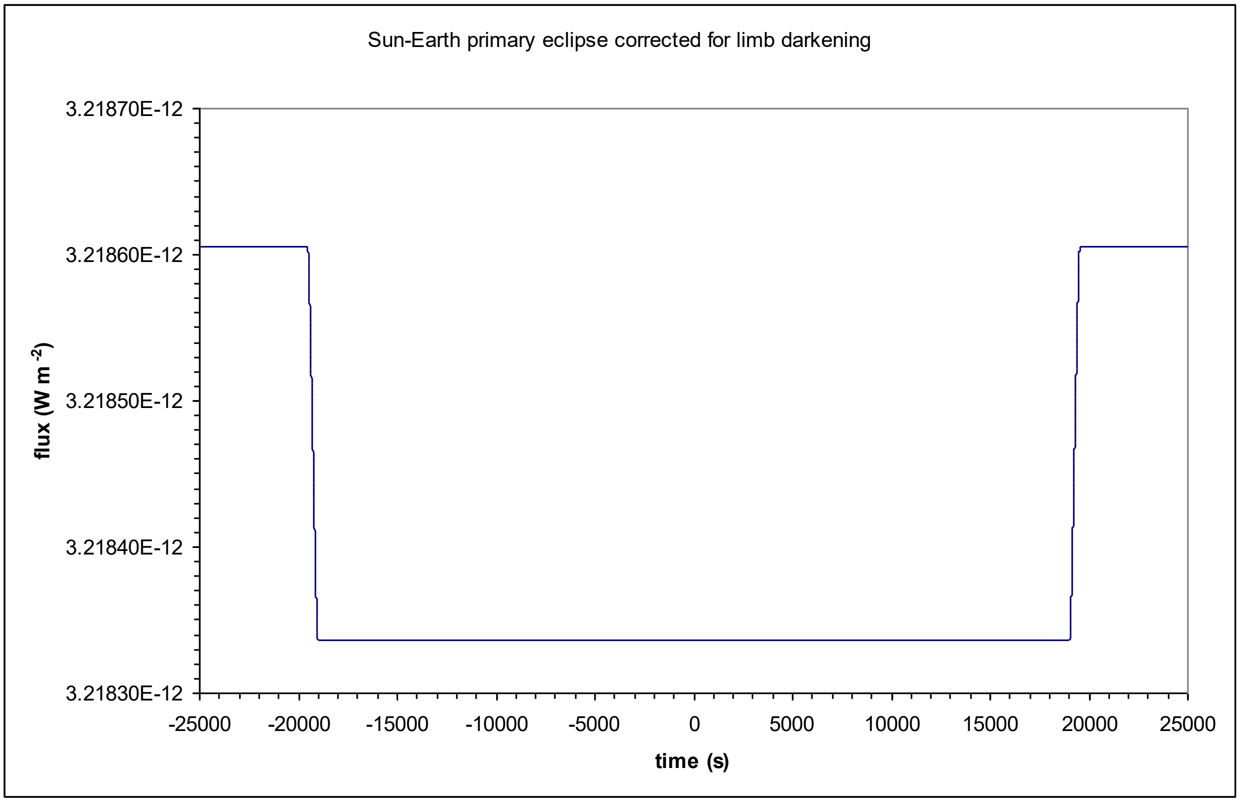


Figure 7. Sun-Earth system without limb darkening.

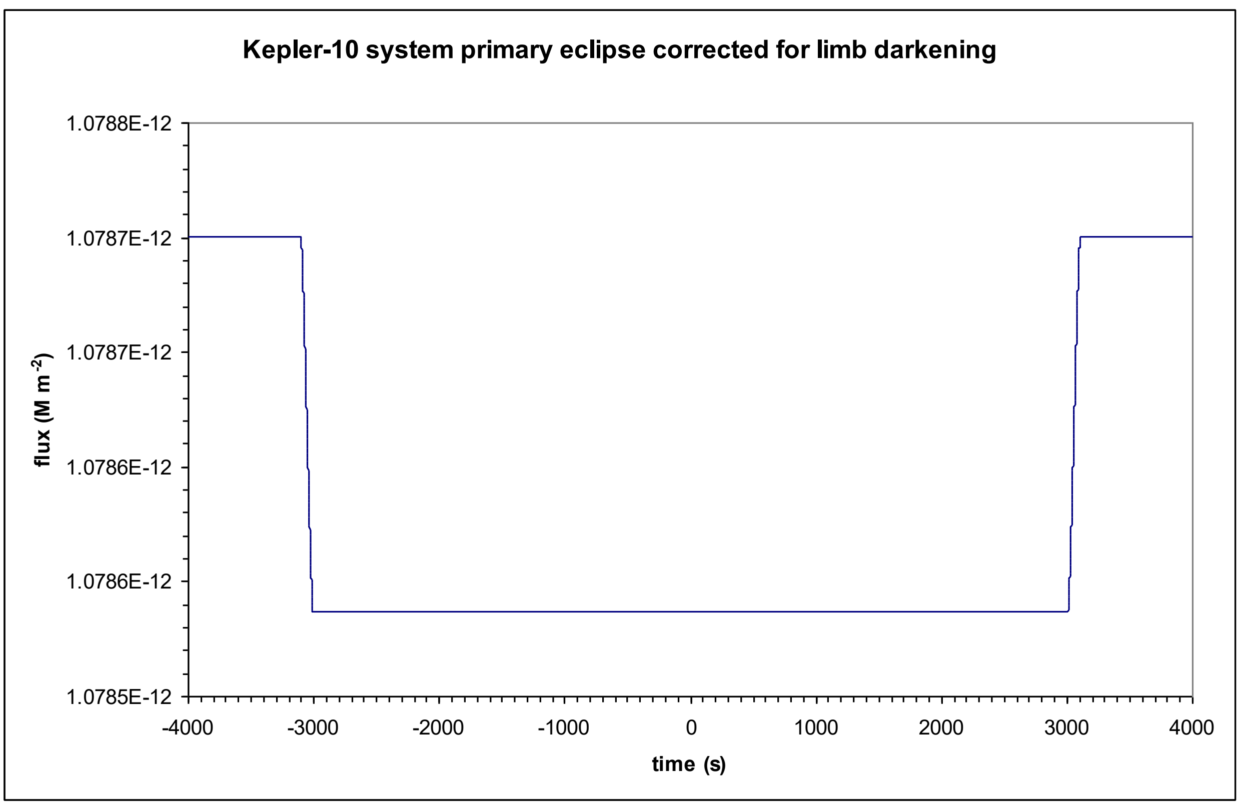


Figure 8. Kepler 10 system after correction for limb darkening

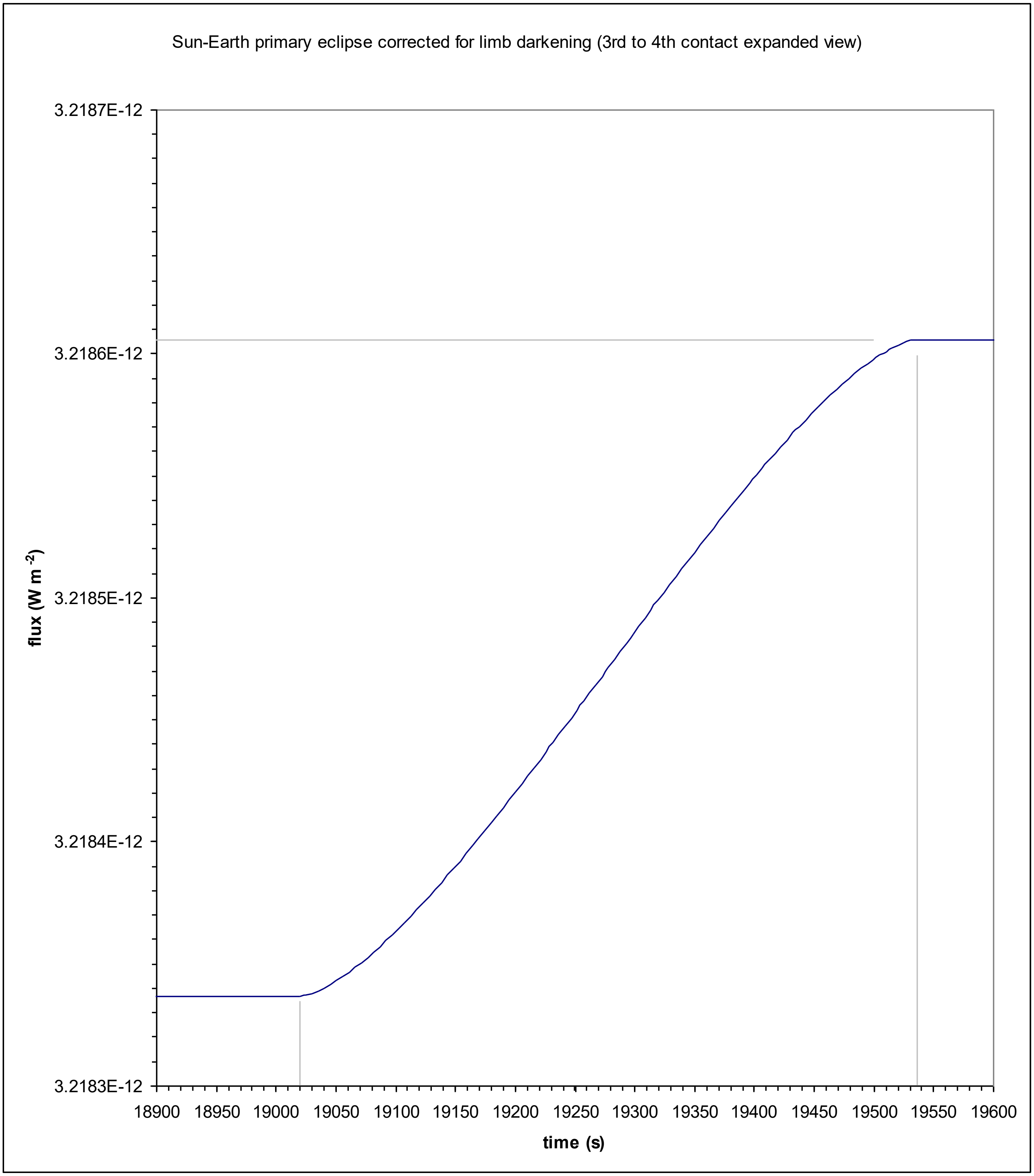


Figure 9. Sun-Earth light curve, expanded view. The third and fourth contacts have guidelines.
Small divisions vertical = 0.00001E-12 W m^-2, small divisions horizontal = 10 s

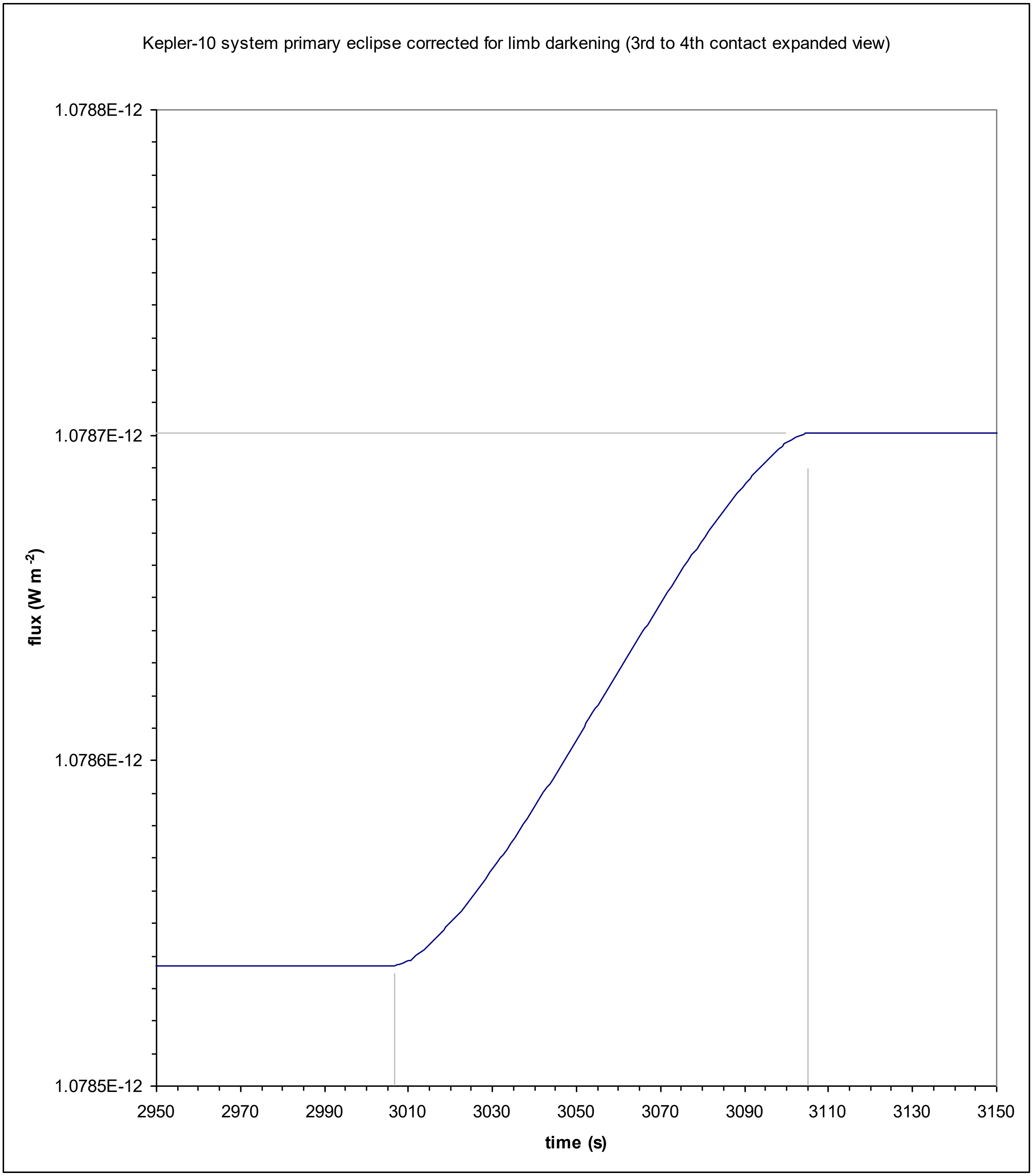


Figure 10. Kepler 10b light curve, expanded view. The third and fourth contacts have guidelines. Small divisions vertical = 0.00001E-12 W m^-2, small divisions horizontal = 5 s

PROCEDURE

1) Set up an EXCEL spreadsheet at shown on page 15. Use a leading single quote mark for the labels. Format given quantities as shown.

2) Measure third and fourth contact times and fluxes using figures 9 and 10; enter them on the spreadsheet, formatting them as "scientific" to the number of decimal places measured.

Cell	Measurement
B16	t3 = third contact time (Sun-Earth); estimate to nearest second
B17	t4 = fourth contact time (Sun-Earth); estimate to nearest second
B18	Fe = flux at time of third contact (Sun-Earth); estimate to 0.000001E-12 W m^-2
B19	Fu = flux at time of fourth contact (Sun-Earth)); estimate to 0.000001E-12 W m^-2
C16	t3 = third contact time (Kepler 10)); estimate to nearest second
C17	t4 = fourth contact time (Kepler 10)); estimate to nearest second
C18	Fe = flux at time of third contact (Kepler 10)); estimate to 0.000001E-12 W m^-2
C19	Fu = flux at time of fourth contact (Kepler 10)); estimate to 0.000001E-12 W m^-2

3) Calculate d = distance in meters (the distances in parsecs have three significant figures)

Cell	Entry	Format
B20	+B12*$B$4	Scientific with two places after the decimal.
C20	+C12*$B$4	"

4) Calculate the surface area of a sphere of radius d. A = 4 π d^2

Cell	Entry	Format
B21	+4*PI()*B20^2	Scientific with two places after the decimal.
C21	+4*PI()*C20^2	"

5) Calculate the flux at the stellar surface. F = σ T^4

Cell	Entry	Format
B22	+$B$2*B13^4	Scientific with two places after the decimal.
C22	+$B$2*C13^4	"

6) Calculate the total stellar power from the observed flux Fu times the area A

Cell	Entry	
B23	+B19*B21	Scientific, with two places after the decimal
C23	+C19*C21	"

7) Calculate the stellar radius from the equation $\text{total stellar power} = (\text{flux at stellar surface}) \times (4\pi R_1^2)$

$$R_1 = \sqrt{\frac{\text{total stellar power}}{4\pi(\text{flux at stellar surface})}}$$

Cell	Entry	Format
B24	+SQRT(B23/(4*PI()*B22))	Scientific, with two places after the decimal
C24	+SQRT(C23/(4*PI()*C22))	"

8) Calculate the planetary radius from the equation $\frac{F_e}{F_u} = \frac{\pi R_1^2 - \pi R_2^2}{\pi R_1}$, $R_2 = R_1 \sqrt{1 - \frac{F_e}{F_u}}$

Cell	Entry	Format
B25	+B24*SQRT(1-B18/B19)	Scientific, with two places after the decimal
C25	+C24*SQRT(1-C18/C19)	"

9) Find the position angles and their cosines at third and fourth contact from the equations $\omega_3 = \frac{2\pi t_3}{P}$, $\omega_4 = \frac{2\pi t_4}{P}$

Cell	Entry	Format
B26	+2*PI()*B16/B14	Scientific, with two places after the decimal
C26	+2*PI()*C16/C14	"
B27	+2*PI()*B17/B14	"
C27	+2*PI()*C17/C14	"
B28	+COS(B26)	"
C28	+COS(C26)	"
B29	+COS(B27)	"
C29	+COS(C27)	"

10) Find the orbital radius a and the inclination angle a from the equations

$$a = \sqrt{\frac{\cos^2\omega_3 \left(R_1 + R_2\right)^2 - \cos^2\omega_4 \left(R_1 - R_2\right)^2}{\cos^2\omega_3 - \cos^2\omega_4}}$$

$$\sin i = 2\sqrt{\frac{R_1 R_2}{\cos^2\omega_3 \left(R_1 + R_2\right)^2 - \cos^2\omega_4 \left(R_1 - R_2\right)^2}}$$

This will be done in steps

Cell	Entry	Format
B30	+B28^2	Scientific, with two places after the decimal
C30	+C28^2	"
B31	+B29^2	"
C31	+C29^2	"
B32	+(B24+B25)^2	"
C32	+(C24+C25)^2	"
B33	+(B24-B25)^2	"
C34	+(C24-C25)^2	"
B34	+SQRT((B30*B32-B31*B33)/(B30-B31))	"
C34	+SQRT((C30*C32-C31*C33)/(C30-C31))	"
B35	+2*SQRT(B24*B25/(B30*B32-B31*B33))	Scientific, with nine places after the decimal
C35	+2*SQRT(C24*C25/(C30*C32-C31*C33))	"

WARNING! If B35 or C35 is greater than 1, you will not be able to continue. Check for EXCEL entry errors or poor values obtained from reading data from figures 8 and 9.

Cell	Entry	Format
B36	+ASIN(B35)	Scientific, with two places after the decimal
C36	+ASIN(C35)	"
B37	+B36*180/PI()	"
C37	+C36*180/PI()	"

11) Find the masses of the star and the planet, M1 and M2, from the equations

$$M_1 = \frac{2\pi\left(2a^3\,\pi\sin i - a^2\,K\,P\right)}{G\,P^2\,\sin i}$$

$$M_2 = \frac{2a^2\,K\,\pi}{G\,P\sin i}$$

The first equation will be done in parts

Cell	Entry	Format
B38	+2*PI()*B34^3*B35	Scientific, with two places after the decimal
C38	+2*PI()*C34^3*C35	"
B39	+B34^2*B15*B14	"
C39	+C34^2*C15*C14	"
B40	+$B$3*B14^2*B35	"
C40	+$B$3*C14^2*C35	"
B41	+2*PI()*(B38-B39)/B40	"
C41	+2*PI()*(C38-C39)/C40	"
B42	+2*PI()*B34^2*B15/($B$3*B14*B35)	"
C42	+2*PI()*C34^2*C15/($B$3*C14*C35)	"

12) Find the density in g cm^-3 for the planets. This will be done in parts.

Cell	Entry	Format
B43	+B42*1000	Scientific, with two places after the decimal
C43	+C42*1000	"
B44	+B25*100	"
C44	+C25*100	"
B45	+(4/3)*PI()*B44^3	"
C45	+(4/3)*PI()*C44^3	"
B46	+B43/B45	"
C46	+C43/C45	"

13) Calculate relative errors in % for the Sun-Earth system (M1, M2, R1, R2, density)

$$\text{Relative error}(\%) = \frac{(\text{measured value} - \text{true value})}{(\text{true value})} \times 100\%$$

Cell	Entry	Format
B47	+100*(B41-B6)/B6	Scientific, with two places after the decimal
B48	+100*(B42-B7)/B7	"
B49	+100*(B24-B8)/B8	"
B50	+100*(B25-B9)/B9	"
B51	+100*(B46-B10)/B10	"

Microsoft Excel - STUDENT.xls

	A	B	C
1	Name	Date	
2	sigma Stefan-Boltzmann Constant (W m^-2 K^-4)	5.670400E-08	
3	G Newtonian gravitational constant (m^3 kg^-1 s^-2)	6.6743E-11	
4	parsec (m)	3.08567758131E+16	
5	AU (m)	1.49597870691E+11	
6	Solar mass (kg)	1.9891E+30	
7	Earth mass (kg)	5.9736E+24	
8	Solar radius (m)	6.96342E+08	
9	Earth radius (m)	6.3710E+06	
10	Earth density (g cm^-3)	5.5150E+00	
11		Sun-Earth	Kepler 10
12	distance (parsecs)	100	173
13	Teff (K)	5778	5627
14	P period of revolution (s)	3.14719821E+07	7.23596E+04
15	K Doppler shift semi-amplitude (m s^-1)	0.09	3.3
16	t3 third contant time (s)		
17	t4 fourth contact time (s)		
18	Fe flux eclipsed star (W m^-2)		
19	Fu flux un-eclipsed star (W m^-2)		
20	d distance (m)		
21	A surface area of a sphere of radius d (m^2)		
22	F flux at stellar surface (W m^-2)		
23	total stellar power (W)		
24	R1 radius of star (m)		
25	R2 radius of planet (m)		
26	omega3 (radians)		
27	omega4 (radians)		
28	cos(omega3)		
29	cos(omega4)		
30	cos(omega3)^2		
31	cos(omega4)^2		
32	(R1+R2)^2 (m^2)		
33	(R1-R2)^2 (m^2)		
34	a (m)		
35	sin i		
36	i (radians)		
37	i (degrees)		
38	2 Pi a^3 sin I (m^3)		
39	a^2 K P (m^3)		
40	G P^2 sin I (m^3 kg^-1)		
41	M1 stellar mass (kg)		
42	M2 planetary mass (kg)		
43	planetary mass (g)		
44	planetary radius (cm)		
45	planetary volume (cm^3)		
46	planetary density (g cm^-3)		
47	Relative error M1 (%)		
48	Relative error M2 (%)		
49	Relative error R1 (%)		
50	Relative error R2 (%)		
51	Relative error planet density (%)		
52			

Sheet1 / Sheet2 / Sheet3

Ready

Questions

1) Compare your results for the Sun-Earth system with the real values (M1, M2, R1, R2, density). How did you do? (Look at the relative errors)

2) Suppose two measured values, 1.2345 and 1.2344. How many significant figures have we?

   Suppose we have to find the difference 1.2345 – 1.2344. How many significant figures have we?

   How many significant figures did you get for the un-eclipsed and eclipsed fluxes for Kepler 10?

   How many significant figures do you get for (flux un-eclipsed)-(flux eclipsed)?

   What significance does this exercise have for the Kepler instrumentation?

3) Could the LGA's have detected Earth using technology equivalent to Kepler?

   Could the LGA's have detected Earth using technology equivalent to HIRES?

   (YES, NO, MAYBE)
   Explain your answers.

4) Assuming a Bond Albedo of 0.1, the temperature of Kepler 10b is 1833K, hot enough to melt iron. Why is it so hot?

   Is the density of Kepler 10b like anything in the Solar System?

GET INVOLVED

The Kepler team uses a computer (IQ=0) to search for planet candidates. Some 75 more candidates have been found by amateurs scanning the publicly released data. So far, about 100,000 volunteers have looked at over a million data sets. The citizen scientist participation has been set up by the zoonivers group. Go to http://www.planethunters.org/