

Binary Systems

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Objective

Investigate binary systems using Kepler's Laws

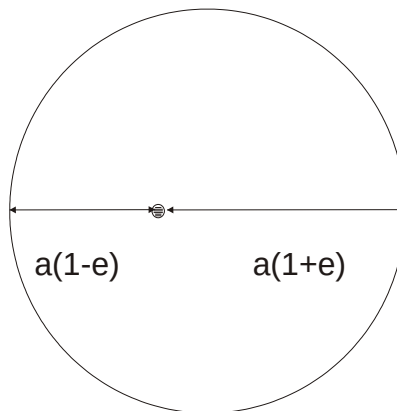
Introduction

Kepler's laws have been applied to distant binary systems for over two centuries. In Newton's general form, we have

- 1) The two objects' orbits are ellipses in the same plane with a common focus corresponding to the center of gravity of the system.
- 2) An object sweeps out equal areas in equal times within its own ellipse.
- 3) $(m_1 + m_2)P^2 = (a_1 + a_2)^3$

where m_1 and m_2 are the masses in solar mass units,
 P is the period in years,
and a_1 and a_2 are the semi-major axes in AU.

Ellipse Geometry

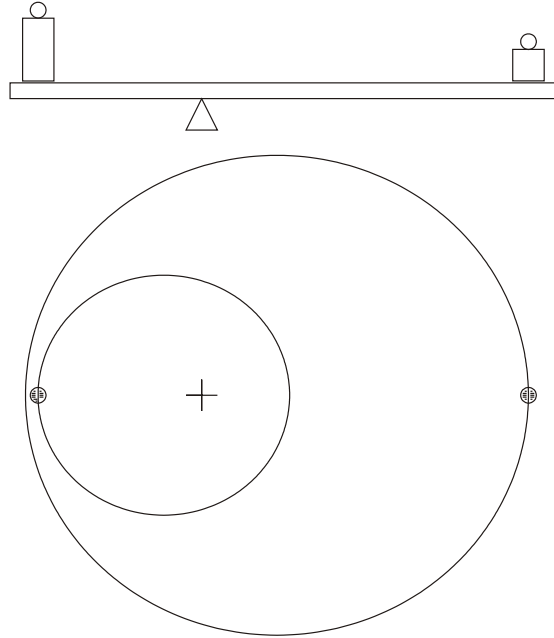


The eccentricity, e and the semi-major axis, a define the ellipse. One focus is shown as a dot.

The major axis = $a(1-e) + a(1+e) = 2a$.

The minor axis (not needed here) is $2a\sqrt{1-e^2}$

In the figure below, the two stars are shown as dots, while a cross marks the center



of gravity. The more massive star or bits in the smaller ellipse. The “see-saw” principle is

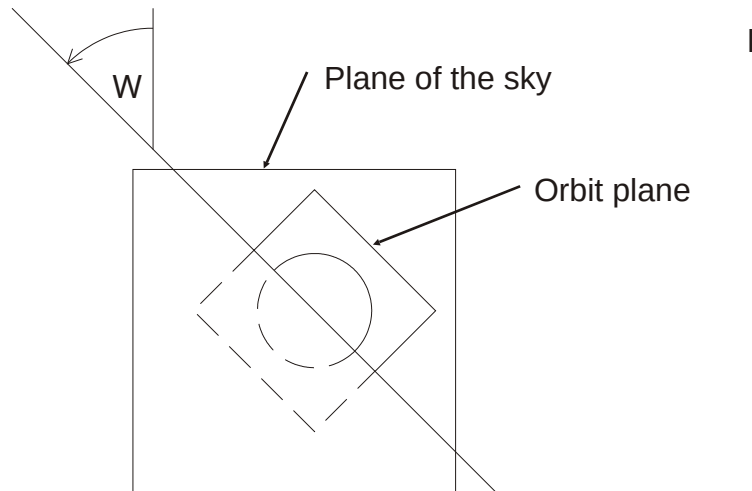
$$m_1 a_1 = m_2 a_2$$

In order to preserve the center of gravity, the periods and the eccentricities must be the same. The stars must go around in the same direction, and the whole figure lies in a plane.

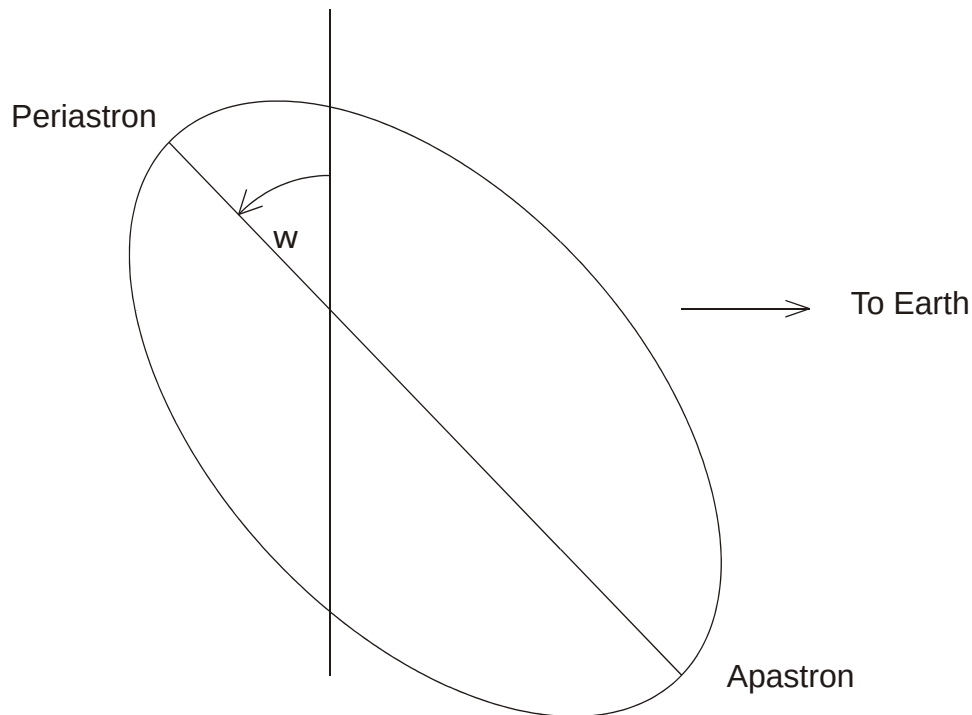
Still more geometry

The figures shown on the previous pages are what an observer in a privileged position would see. The Earth is not in a privileged position, and what an observer on Earth sees is a version of the figure that has been rotated, twisted and inclined. There are three angles that have to be determined before the system can be completely understood.

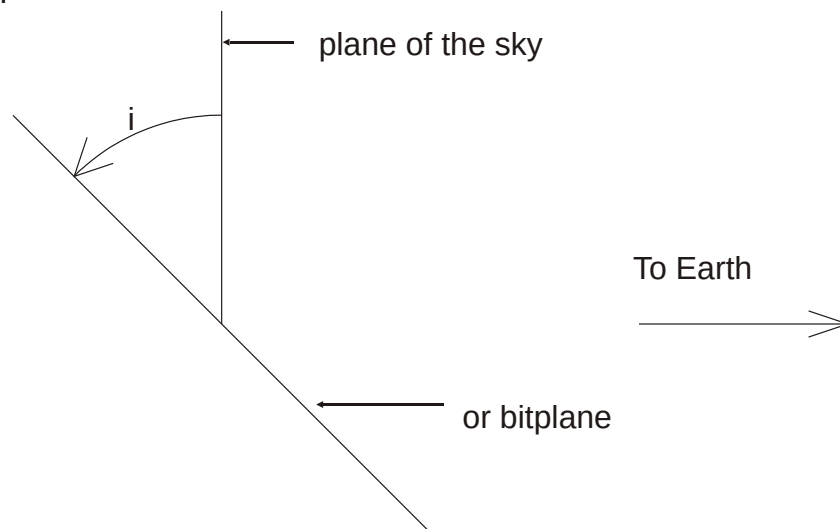
The plane of an orbit intersects the “plane of the sky” along a line, the line of nodes. The position angle of the node, measured from North has the symbol W .



The longitude of periastron is the angle between the line of nodes and the periastron. The symbol is w .



The inclination is the angle the orbit plane makes with the plane of the sky and has the symbol i . When i is 0 degrees the orbit is seen face on; when i is 90 degrees the orbit is seen edge on.



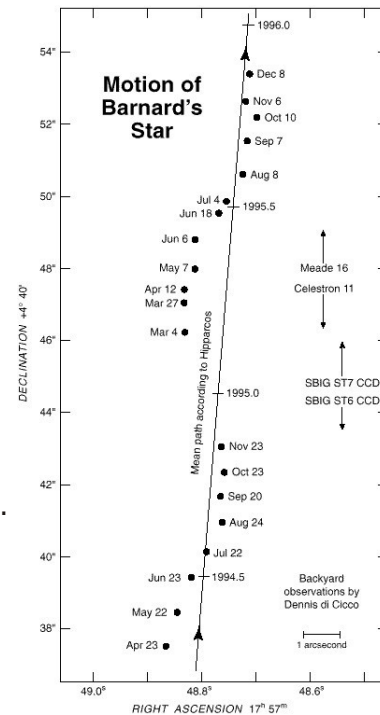
Types of Observations of Binary Systems

The oldest method of double star measurement is to visually measure the position angle and distance between stars in seconds of arc. High quality observations are available from the early nineteenth century on. We must rely on this early work when P is large. Note that what is obtained is a relative orbit. This turns out to be an ellipse, but the focus of the ellipse is not located at the primary star.

A second method is astrometric, where the motions of both stars are found photographically by referring to faint distant stars. This method yields both α and δ . Care must be taken to allow for the motion of the Earth around the Sun, and the relative motion of the Binary system with respect to the Sun also. These complications are shown to the right for Barnard's star, (not a binary system). A large proper motion and a wobble with a 1 year period due to the Earth's motion are both evident.

A third type of observation of a binary system is spectroscopic, using the Doppler shift of a star's spectrum lines to calculate the star's velocity component in our line of sight. Note that if the inclination angle is zero, the orbits are seen face on, and no periodic Doppler shifts are seen.

Finally, if the system has an inclination near 90 degrees, so that we observe the system nearly edge on, we may be able to observe eclipses.



Two points must be made at this stage:

- 1) None of the above methods, taken alone, is sufficient for complete characterization.
- 2) Even if we can't completely characterize a system, we can still deal with it.

Program Instructions

- 1) Start the program according to instructions that will be given to you.
- 2) Enter your name(s) in the box provided. This information will be used for printing.
- 3) From the File menu, select Experiment. This opens a list of binary systems.
- 4) Select a system for study according to instructions that will be given to you.
- 5) To exit the program, go back to the File menu and select Exit.

: If a printer is not available, simply copy the data onto the data sheets directly.

24 Aquarii

This is a typical visual binary system. The relative orbit was observed between the years 1890 and 1932, and these observations have been plotted on the picture of the apparent orbit. They seem like they are of very poor quality, but they represent the best quality possible. The apparent separation is less than one second of arc for all observations. The apparent orbit parameters are

Period	51.33 years
Semi-major axis	0.525 arc seconds
Eccentricity	0.9102
Longitude of Periastron	87.35 degrees
Inclination	56.02 degrees
Node angle	4.95 degrees

Note that the semi-major axis is given in arc-seconds rather than in AU. What is missing is the scale. A recent determination of the parallax (Hipparcos) gives a distance of 43.23 parsecs, which enables us to completely characterize the apparent orbit. The most that can be found without further information is the sum of the masses.

To start the experiment, click on File, then on Experiment, then on 24 Aqr. All of the parameters have been given to you except for the sum of the masses, which it is your job to find. Vary the sum of the masses until your calculated orbit fits the observations. To do this, enter a mass, and then left button mouse click on the OK button. Repeat until satisfied, then fill out the data sheet.

Also, on the data sheet are instructions for converting the semi-major axis from seconds of arc to AU, and the use of Kepler's third law to give a sum of the masses. Fill this out with the help of a calculator.

Data sheet for 24 Aqr

Name _____ ID # _____ Date _____

From the computer simulation: sum of masses: (in solar masses) _____

semi-major axis (in AU) _____

Sirius

Sirius is a very close system and has been studied by a variety of methods. All parameters are well known. The only difficulty is the observation of the tiny secondary, the white dwarf Sirius B, in the glare of the primary, Sirius A, the brightest star in the sky.

Computer simulation: Leftbutton click on the File menu, then the Experiment menu, then Sirius. All orbital parameters, except the masses and the semi-major axis, are shown. Your job is to find both masses.

Make guesses for both mass values, so as to fit the [synthetic] observations, using m_1 for the larger of the masses and m_2 for the smaller. You should be able to get results that are accurate to about 10%, which is the literature value of the uncertainty.

Data sheet for Sirius

Name _____ ID # _____ Date _____

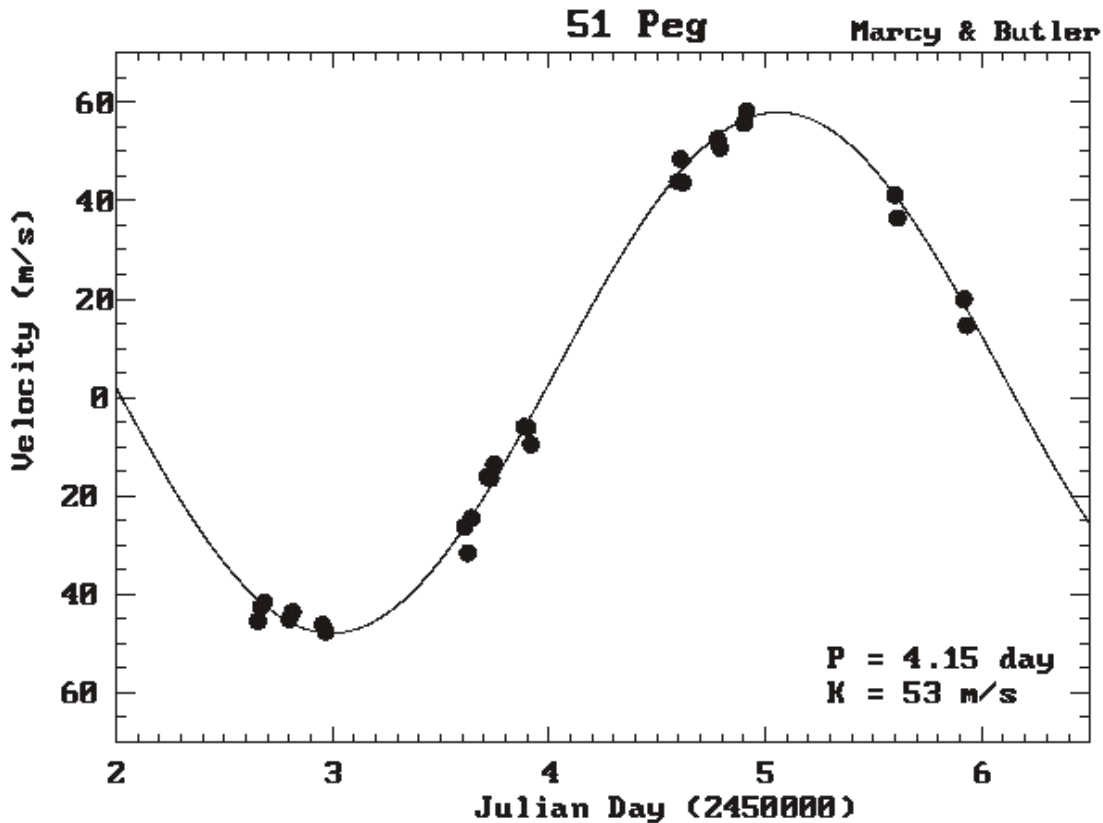
From the computer simulation:

m_1 in solar masses _____ m_2 in solar masses _____ semi-major axis in

AU _____

51 Pegasi

This system consists of a very ordinary, fairly nearby star and what is apparently a planet. The planet is not directly observed, but is inferred by an oscillation in the velocity curve, shown below.



A further refinement on the period, made possible by observation over a longer period of time, gives $P = 4.2293$ days. From the spectral type, we can infer a mass of the primary of about one solar mass. From the shape of the velocity curve, we learn that the eccentricity is zero (or very close to zero). That's it. No more available information.

Since we have a round orbit, the problem is greatly simplified. We have to guess a value of the inclination angle and then find a value of the secondary mass that causes the above velocity curve in the primary. Any value of i is allowed except exactly zero (if the orbit were exactly face on, there couldn't be any velocity curve).

Computer simulation: Left button click on the File menu, then the Experiment menu, then 51 Peg. Your job is to try values of the inclination (20, 50 and 80 degrees), and

for each, find a value of m_2 that gives the correct velocity maximum and minimum of 0.053 km/sec.

Data sheet for 51 Peg

Name _____ ID # _____ Date _____

From the computer simulation:

Inclination angle i , degrees (au)	m_2 (solar masses)	semi-major axis
20	_____	_____
50	_____	_____
80	_____	_____

For the following calculation, assume an inclination of 80 degrees. A calculator is needed.

Convert m_2 from solar masses to Jupiter masses

$$m_2 \text{ solar masses} * \frac{1047.355 \text{ Jupiter masses}}{1 \text{ solar mass}} = (\quad) * 1047.355 = \underline{\hspace{2cm}}$$

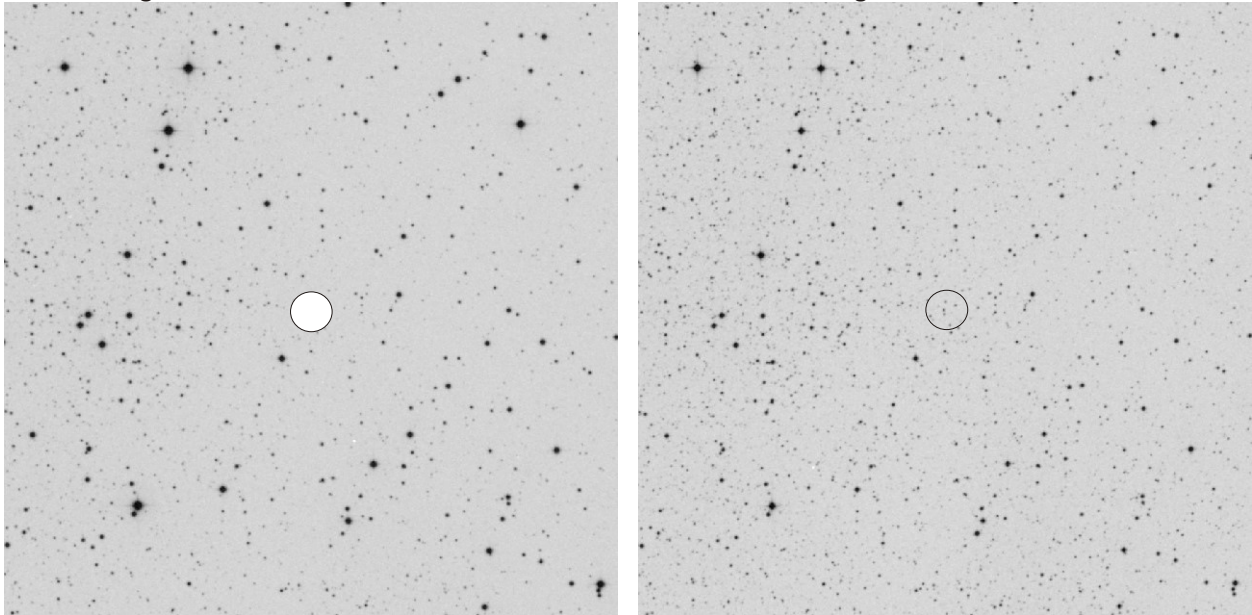
Questions

- 1) What happens to the calculated mass m_2 as the inclination becomes smaller?
- 2) Could the mass m_2 increase without limit, say, up to 10 solar masses, and still be a planet?
- 3) The mean distance of Mercury from the Sun is 0.387 AU. Given that 51 Peg is similar to our Sun, is this planet hotter than mercury?

V404

Here we have a mystery to solve with insufficient data. Here is what is known:

V404 is a very faint object in the constellation Cygnus. It is currently around $m=18$ and fading. It is in the same position as Nova Cyg 1938, which at maximum brightness was at $m=11$. It has been active since that time, and emitted a burst in the X-Ray in 1989, detected by the GINGA satellite. Within days of detection of the burst, the visual magnitude was about 12. The quiescent magnitude is 19-20. Immediately prior to the X-Ray burst, a plate taken to look at something else contained V404, and the measured visual magnitude was 18.3. A radio burst was detected during the 1989 burst as well.



Here are some pictures taken with the Palomar 48 inch Schmidt. The scale is 15' on a side. Pictures are shown as negatives. Left: blue sensitive plate, June 6, 1989. Right: red sensitive plate, Sept. 9, 1991. V404 is at the center of the circle. The change in brightness is evident.

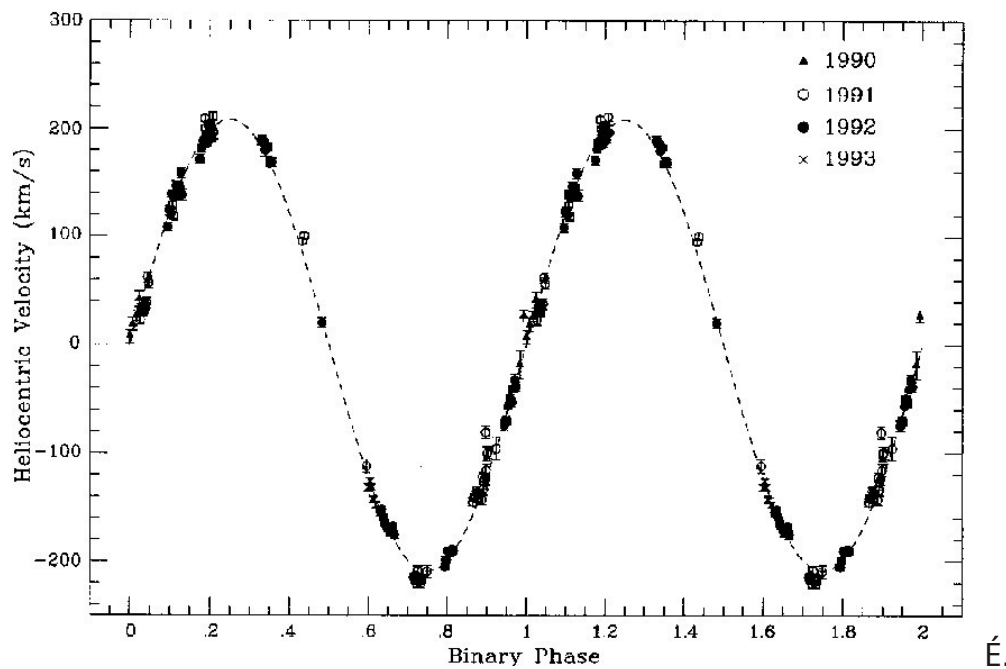
Spectra taken in early June of 1989 reveal strong emission lines of hydrogen and helium positive ion, indicative of very hot gas. Later spectra show a fairly normal star, of type K0 IV, but with absorption lines that are partially filled in with emission.

What have we got? We have a cool subgiant star plus some very hot gas. The gas is being lost by the star, and then getting heated up somehow. One type of system like this

is known, and calls for a binary system, with the second body invisible, but surrounded by an accretion disk of gas. The gas is lost from a tidally distorted star and heats up by friction within the accretion disk. Such disks of gas are not very stable, and sometimes a lot of gas collapses inward, explaining the variability.

The next thing to look for is rotation of the system by searching for Doppler shifts in the spectrum of the star. We indeed find a spectroscopic binary system, and the velocity curve is shown below [J. Casares and P.A. Charles, Monthly Notices of The Royal Astronomical Society, 271, L5-L9 (1994)]

The velocity peaks are an extraordinary 208.5 km/sec. The period is 6.4714 days. The sine wave shape of the velocity curve indicates that the eccentricity is zero. The inclination cannot be determined from the data, except to note that there are no observed eclipses of the accretion disk, which limits the inclination to less than 80 degrees.



The mass of the star that we see has to be guessed at. The spectrum is that of a K0 sub giant, and by looking at eclipsing binaries containing that type of star, we find that the mass should be 1.3 solar masses. However, we are looking at a stripped K0 sub giant that has lost an unknown amount of its mass, and is continuing to lose mass. The current guess is 1 solar mass with a large uncertainty. This star will be the secondary, and the invisible mass will be the primary. Since it is not seen, the primary must be a compact object.

Compact objects start with white dwarfs. They are cinders of stars that have burned all their hydrogen fuel and are not massive enough to burn helium. There is a firm limit to

their mass, the Chandrasekhar limit of about 1.4 solar masses. If the mass is higher than this, a white dwarf cannot exist, and collapses. What appears to happen is that all the mass is converted to neutrons, and the whole star becomes a giant atomic nucleus. This is a neutron star. Neutron stars can exist up to about 3 solar masses, before they too, collapse under their own weight. What happens then is unknown. We know of no mechanism that would halt the collapse down to a dimensionless point. We do know something about a lot of mass in a very small volume: the escape velocity exceeds the speed of light and we have a black hole. These are the classes of possibilities you will test for in a computer simulation.

Computer simulation: Left button click on the File menu, then the Experiment menu, then V404. Your job is to try values of the inclination (20, 50 and 80 degrees), and for each, find a value of m_1 that gives the correct velocity maximum and minimum of 208.5 km/sec.

Data sheet for V404

Name _____ ID # _____ Date _____

From the computer simulation:

inclination angle	m_1 (solar masses)	semi-major axis (AU)
20 degrees	_____	_____
50 degrees	_____	_____
80 degrees	_____	_____

Question

What kind of object is this (white dwarf, neutron star or black hole)?

Explain your answer.

Alpha Corona Borealis

This nearby system is an eclipsing binary. It has been studied by a variety of methods and is completely characterized. The star properties are given below

	Primary	Secondary
spectrum	B9.5 V	G5 V
effective temperature (K)	9700	5790
mass (solar masses)	2.58	0.92
radius (solar radii)	3.04	0.90
luminosity (Sun=1)	74.1	0.81

The distance to the system is 22.9 parsecs, based on a good parallax (Hipparcos).

Computer simulation: Left button click on the File menu, then the Experiment menu, then alpha Cr B. All parameters have been entered. Left click on the OK button to get things started. Your job is to watch the simulation and to find out which eclipse is which.

Data sheet for a Cr B

Name _____ ID # _____ Date _____

There are two eclipses:

- a) The secondary is behind the primary
- b) The secondary is in front of the primary.

The deeper eclipse has a depth of 0.1 magnitudes, and the shallower eclipse has a depth of only 0.02 magnitudes.

Questions

Which eclipse is which?

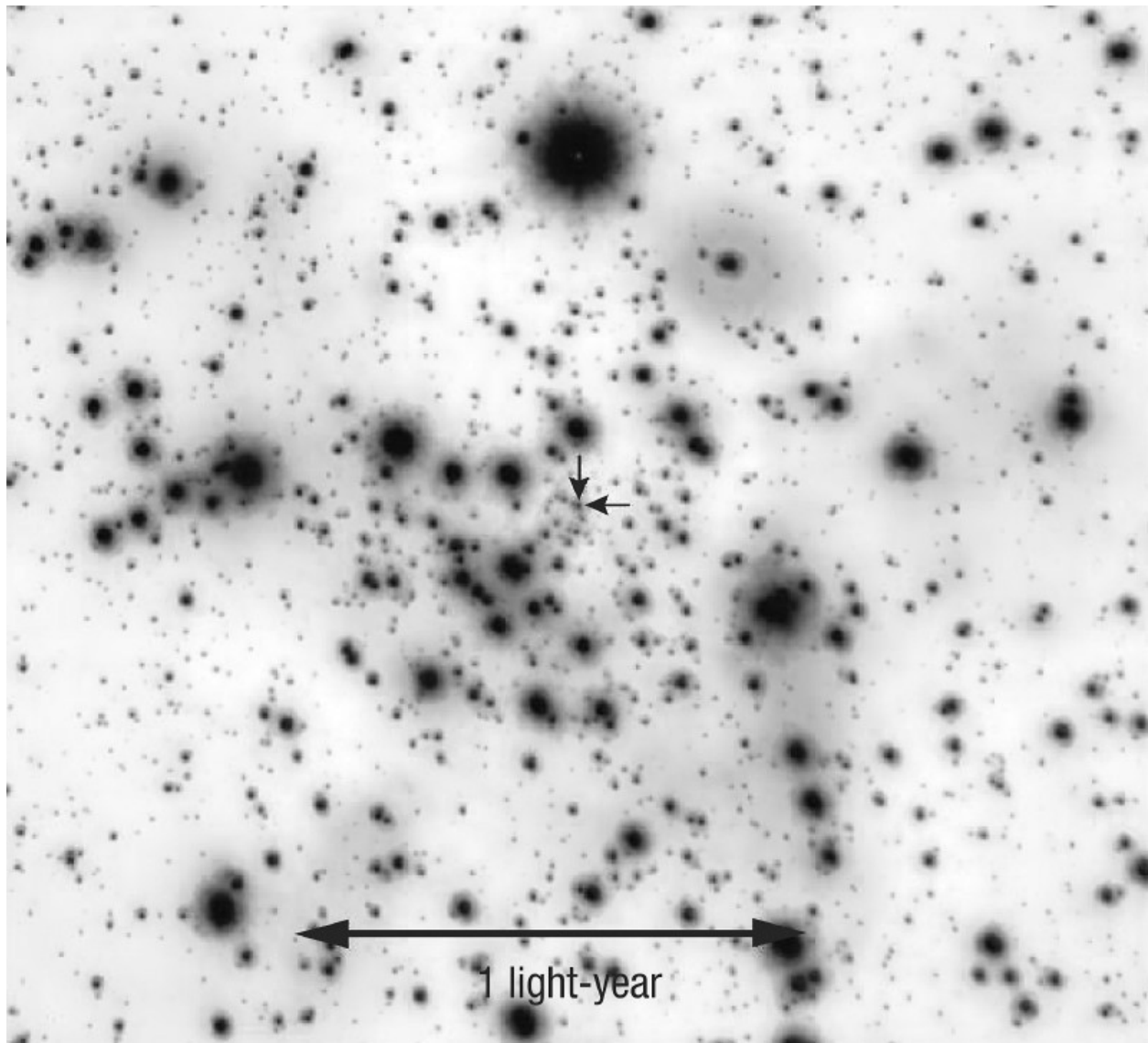
Explain the relative depth of the eclipses. (Hint: for both eclipses, the area that is hidden is identical).

Sgr A*

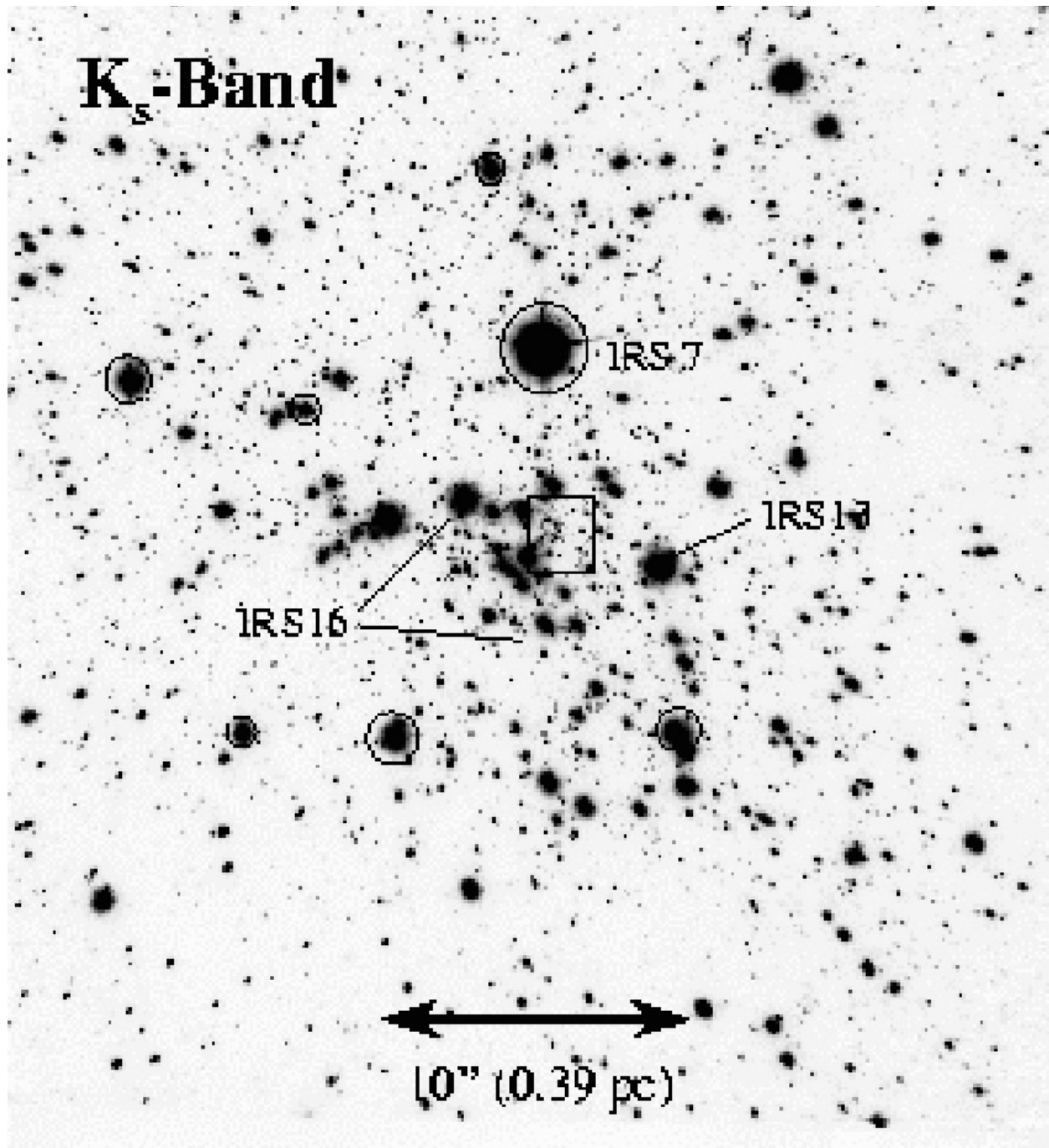
The strong radio source Sagittarius A* lies at the very center of the milky way galaxy. A radio image, shown below, reveals a dramatic, whirlpool-like structure.



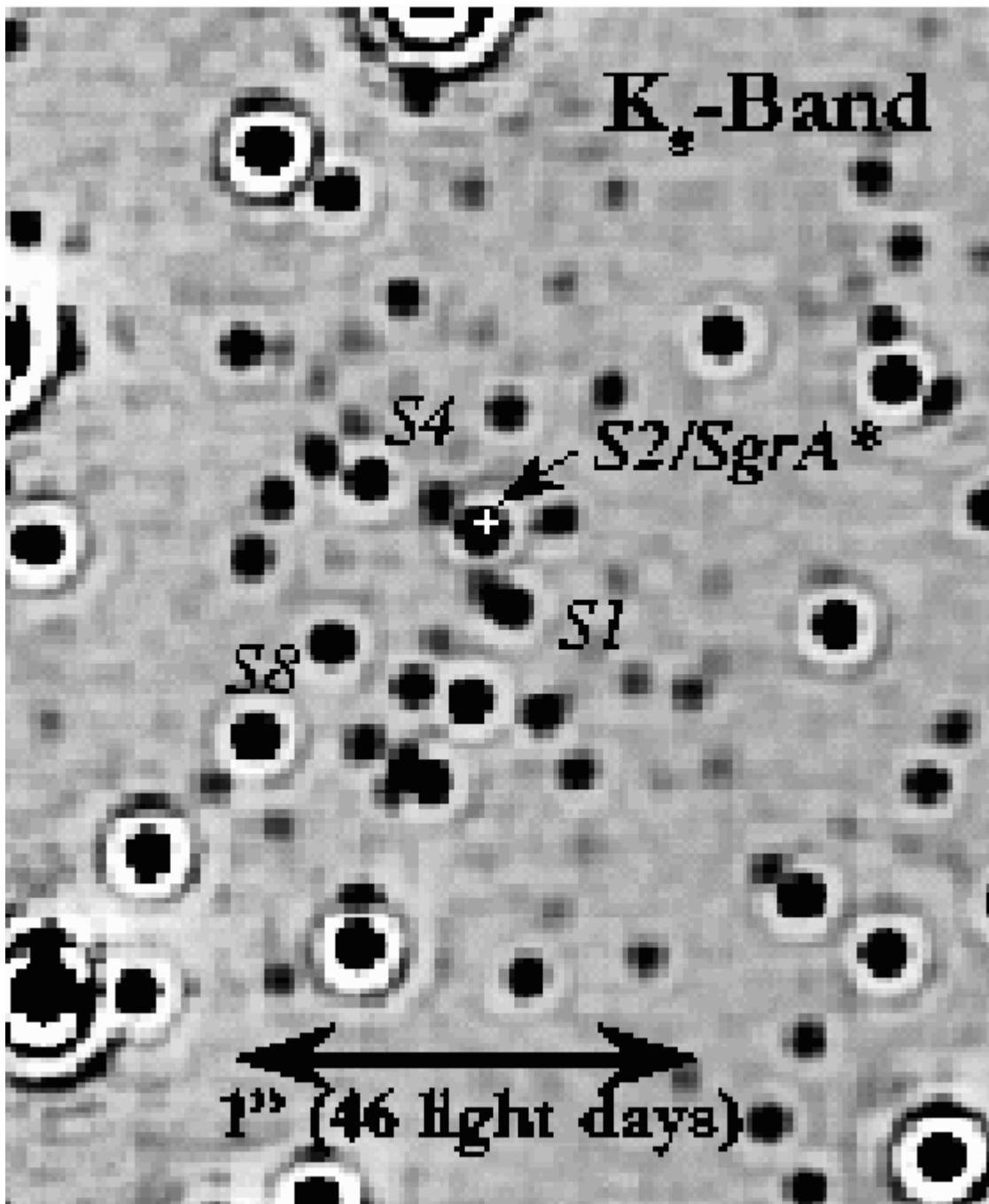
The region is also a source of X-Rays, suggesting in-fall of matter onto a massive, compact object. A picture of the galactic center in X-Rays from the Chandra orbiting observatory is shown below.



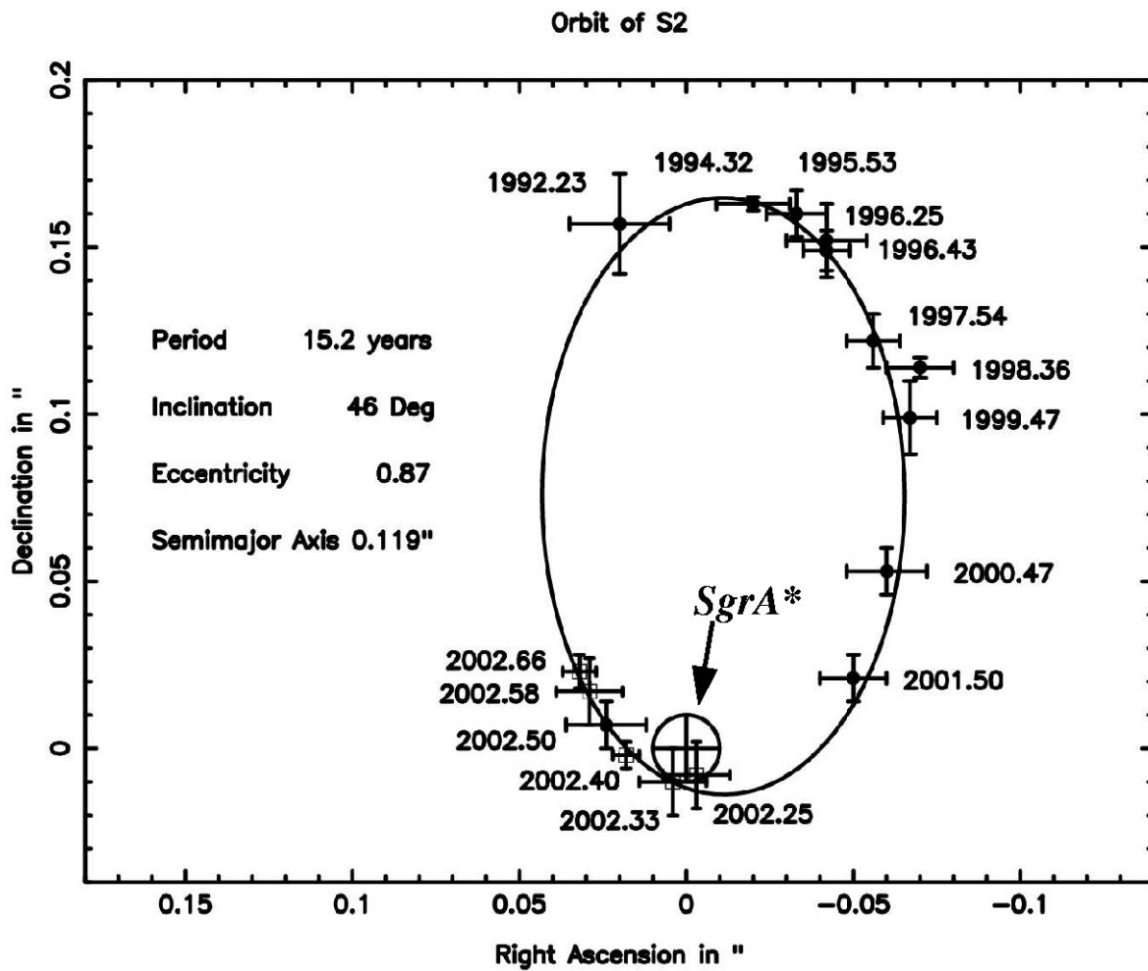
The galactic center region contains a lot of dust, and it is hidden from us in visible light. We are able to see through the dust at infra-red wavelengths, utilizing the atmospheric windows that are relatively transparent. An infra-red view of the region, photographed with the European Space Agency's Very Large Telescope in northern Chile is shown below.



There are thousands of stars in a small space. By comparison, the width of the photograph is about half of the distance between the Sun and the nearest star. The dust is heated up by the stars, and glows by blackbody radiation (dark in this negative view). Arrows point to the galactic center. Thanks to advances in adaptive optics we are able to zoom in on the central region:



A break through paper on Sgr A* and the stars near it appeared in Nature on October 17, 2002. The backup research paper was submitted the following day to the Astrophysical Journal by Andrea Eckart et.al. The star labeled S2 in the picture on page 17 passed very close to the unseen primary in the spring of 2002, and its orbit was determined. The orbit and the 16 measured positions of S2 are shown in the figure below.



In order to convert angular measure to astronomical units we need a distance to the galactic center. This distance is 8 kiloparsecs, and is not known more accurately than one significant figure. There is also an astrometric error in the measured positions of S2, as indicated above by the large error bars. A third source of error is matching the infra-red positions of S2 with the accurately known radio position of Sgr A*, an error which has now been reduced to 0.010 arc-seconds.

Beside beating the competition, there was good reason for publishing early, and that is the importance of this work. The close passage of S2 to the primary (about 124 AU, three times the distance from the Sun to the planet Pluto) places a constraint on the size of the central object. Since S2 didn't crash, Sgr A* has a radius smaller than about 124 AU, and this eliminates every idea brought forward of what the central object can be except for a black hole.

Computer simulation: Left button click on the File menu, then the Experiment menu, then Sgr A*. All parameters for the orbit of S2 have been entered, except for the mass of the primary, M1

Guess a value for M1 and then left click on the OK button to get things started. Repeat until you have a satisfactory fit to the ellipse based on the 16 data points. At this point, the screen view of the apparent orbit will look something like the published figure on the previous page. Print the result.

Data sheet for Sgr A*

Name _____ ID # _____ Date _____

Mass M1 in solar masses _____

Question:

Einstein aside, how do you know that Sgr A* isn't a single, extremely massive star?
(Hint: look at the photo graphs)